Everything and More: A Compact History of Infinity

David Foster Wallace

November 2004 / paperback / ISBN 0-393-32629-2 / 336 pages /$14.95
2003 / cloth / ISBN 0-393-00338-8 / 320 pages / $23.95

David Foster Wallace is a high-profile American writer, important some would say. I confess, I'm a Wallace fan, though I'm waiting to see if this relative youngster (somewhere around his fourth decade) maintains the virtuosity and expansiveness of his greatest book so far, *Infinite Jest*. To be sure, he has all the markings of a great writer: ambition, intelligence, a sense of gravitas, and, perhaps most notably, style. He has achieved a brand recognition that guarantees him regular appearances in the pages of slick gazettes like *The New Yorker* and *Harper's*. He is youthful, boisterous and charmingly irreverent, employing trademark devices that strategically dazzle and confuse the reader, while seldom failing to entertain. He writes with an earnestness and self-awareness that is almost as compelling as the story he is telling. He is simply fun to hear on the page, regardless of the topic. And every so often he lands a sublime observation with such clarity that it is breathtaking. So when I heard that this talented writer had published a book on mathematics, I felt like the pimply weakling in school who is tapped on the shoulder by the popular senior and asked to sit at his table in the lunchroom while he tells his pals all about how cool I am. I was eager to see my profession through the eyes of a charming admirer.

Wallace splits his time between fiction and nonfiction. His first collection of essays *A Supposedly Fun Thing I'll Never Do Again* appeared shortly after *Infinite Jest* and helped solidify his status as a literary celebrity. Tennis, popular culture, and science are frequent sources of inspiration. There aren't many serious writers working today who find something universal in cross-court volleys; neither are there many who use mathematics as artistic media. In one of the first essays in *A Supposedly Fun Thing I'll Never Do Again*, "Derivative Sport in Tornado Alley", he describes his edge in competitive junior tennis: "'Unless you're one of those rare mutant virtuosos of raw force, you'll find that competitive tennis, like money pool, requires geometric thinking, the ability to calculate not merely your own angles but the angles of response to your angles. Because the expansion of response-possibilities is quadratic, you are required to think n shots ahead, where n is a hyperbolic function limited by the sinh of the opponent's talent and the cosh of the number of shots in the rally so far (roughly)." This passage (which, incidentally, gets a more refined treatment in *Infinite Jest* as James Incandenza's philosophy of tennis) gives a hint about Wallace's technique. As a writer, he is not a mutant virtuoso of raw force, and, indeed, he does calculate a kind of triangulation between his reader, his editor, and his subject matter. But he ably scores points with his readers, whatever the artifice. While there is an element of high jinks that runs throughout Wallace's work, he is deeply concerned about the fate of humanity and, in particular, humanity in thrall to the miracles of technology, some of which he uses to keep his readers enthralled.

In many ways, Wallace was a natural choice to kick off a new series by Atlas Books and Norton & Co. celebrating for a popular audience some of the great achievements of science and mathematics. Technical details figure prominently in his essays and fiction and he has repeatedly professed an affection for mathematics. In that respect E&M is a curious love letter to the discipline, one that I suspect he has long wanted to write.

There is a math legend, occasionally reinforced by fact, of the PhD student who writes a lengthy body of theory built upon assumptions that are satisfied only on the empty set. One senior colleague remarked that most of the obvious mistakes occur at the very beginning. While Wallace's mistakes are numerous, easily spotted, and by now well documented, the beginning of E&M is pretty much right on the money.

It's difficult to say, though, exactly where the book begins. The editor seems to think that it begins on page 1, which Wallace calls a "Small But Necessary Forword." This chapterette is indeed in the best tradition of forwords: Wallace addresses his readers directly in an informal tone and muses about the challenges of writing about mathematics in a way that is interesting and relevant to non-mathematicians. The rest of

the book, however, continues in this voice, addressing his editor and the reader directly, frequently drawing attention not only to the fact that you are reading, but he is writing a book about mathematics marketed, as Wallace would say, by a swank publishing house in New York. Wallace expertly applies this type of narrative subversion. If you get a thrill out of seeing the institution of The Author turned on its ear, then you are in for a treat with E&M – this form of iconoclasm is almost unheard of in technical writing.

Despite the editor's pagination, Wallace begins *his* book with §1*a* on page 5. From here on out his headingless numbered § is the only hint the reader will get about major transitions of topic. If someone wishes to argue that he is parodying technical writing [1] then here is exhibit A. There are a whole zoo of subheadings and parenthetical blind alleys running though the narrative. These are unique to the book and are variously denoted by **EMERGENCY GLOSSARY** and **EMERGENCY GLOSSARY I W/ AN ASSOCIATED NARRATIVE TIME-JUMP** and **IN CASE YOU'RE INTERESTED** (**IYI** for short) and **DEFINITELY IYI** and **INTERPOLATIVE IYI** and **INTERPOLATION** and **MINI-INTERPOLATION** and **IYI-GRADE INTERPOLATION** and **UNAVOIDABLE BUT ULTIMATELY IYI-GRADE INTERPOLATION** and **ADMINISTRATIVE INTERPOLATION** and **QUICK FOREST-V.-TREE INTERPOLATION** . . . to name a few.

In the first numbered section of the book, Wallace stakes out his territory by taking aim at serious historians of mathematics on one flank, and modern pop writers on the other. A quotation from Eric Bell [*Men of Mathematics* (1937)] is all the evidence Wallace needs to wash his hands of serious math history, which he dismisses with the curt introductory remark, "There is such a thing as an historian of mathematics." I don't think he has anything per se against historians of mathematics, though he takes it as a given that most of his readers would sooner get a spinal tap than read a straightforward mathematical history. He takes a little more time with modern pop writers, however, on the correct assumption that his readers will have seen a Hollywood movie or maybe even have read a popular history that boils mathematicians down to messy hair, bad fashion, and incomprehensible concerns that lead inexorably to social faux pas – what Wallace calls "nerdy little bowtied fissiparous creatures" in probably the first, and hopefully not the last, such use of *fissiparous* (see glossary). He takes particular issue with the archetypal caricature of "The Mentally Ill Mathematician". While right on the mark, he can't entirely claim the high ground on this point. James O. Incandenza, one of the shadowy central characters in *Infinite Jest*, is an optical physicist who commits suicide by cooking his head in a microwave oven. Still, Wallace admirably steers around these templates while driving just close enough for us to see the wreckage through the ambulances. He admits that the central character of E&M, Georg Cantor, makes it very tempting to play to these stereotypes, but he argues in a footnote that "the view of ∞ as some forbidden zone or road to insanity – which view was very old and powerful and haunted math for 2000+ years – is precisely what Cantor's own work overturned." And so we find, in a footnote, the inspiration for this loving, 319 page dramatization of mathematics.

For those who don't read the footnotes[2], Wallace restates the thesis in the main text: "Merely knowing about Cantor's accomplishments is different from appreciating them, which latter is the general project here and involves seeing transfinite math as kind of like a tree, one with its roots in the ancient Greek paradoxes of continuity and incommensurability and its branches entwined in the modern crises over math's foundations – Brouwer and Hilbert and Russell and Frege and Zermelo and Gödel and Cohen et al." Wallace drops the names to disarm skeptics who might doubt his qualifications, but his sympathies lie with his less math savvy readers, whom he generously reassures in the very next sentence: "The names right now are less important than the tree thing, which is the main sort of overview-trope you'll be asked to keep in mind." This is quintessential Wallace, playing to the balcony and the pit. Another recognizable device are the footnotes, about which much has been written.[3]

The book, from its inception, is fraught with peril. It's one thing to chronicle the pursuit of one's passions, no matter how arcane or trivial, but to peddle the object itself to the unconverted is very tricky business. Any mathematician who has tried to get beyond the blank stare or the "math was my worst subject" dismissal in casual conversation has an intimate knowledge of the disconnect between our profession and popular perceptions. Enter Wallace, a writer with a rudimentary background in the subject, who proposes not only to explain the significance and historical evolution of a slippery notion, but who also attempts to lead readers through one of the first statements, together with an elementary proof, of this notion as a mathematical object. The latter he means to accomplish for the benefit of readers whom he assumes are

[1] As if his footnotes weren't obvious enough (see below).

[2] Obviously not you.

[3] If this is your first encounter with David Foster Wallace, then all you need to know is that about half of his texts are contained in footnotes which are entertaining and maddening at the same time. Since the observant reader will, by now, have gotten the point, I will desist with my cheap imitation.

not familiar with induction or proof by contradiction much less mathematical "proof". It may be arrogance, ignorance, enthusiasm or, more likely, all of these that sustain him in this labor. The perplexing issue is what sustains the reader.

Infinity shares the same place in the popular imagination as chaos, and these two are perhaps unique among mathematical objects in their mystique and glamour. Since chaos is a bit more difficult to define without an understanding of dynamical systems, Wallace's choice of infinity as a topic is natural though not particularly novel. There has been a steady trickle of books on this topic for all audiences. The most recent of these just came out under the title of *A Brief History of Infinity* by Paolo Zellini and translated by David Marsh (2004). My freshman mathematics Teaching Assistant even gave his rendition in a 45 minute demonstration of the uncountablility of the reals that began with the familiar question "If God is omnipotent, can he create a rock that he cannot lift?" This schoolboy found it quite pleasurable to learn how mathematics could get a handle on such a fancy word. And I still get shivers when I hear or say the German "unendlich". Wallace's presentation, though largely from the hip, is much more entertaining and expansive than my TA's. It is primarily geared towards the average university educated humanities major, but Wallace has a message for the technically inclined too: that mathematical thinking tends to lead one to trivialize the underlying philosophical problems, and by extension to denude the human experience of much of its richness.

E&M follows a fairly well trodden, if nonlinear, path through the story of infinity. This begins, as the story goes, with Zeno's paradoxes and Eudoxus' theory of proportions, follows the building pressure to resolve the fuzziness of infinitessimals and infinitary arguments from the work of Archimedes through Dedekind with brief stops along the way to mention major and minor mathematicians and scientists in an almost kaleidoscopic whirl, until we reach the completion of the historical dialectic that Georg Cantor represents, and upon whom Wallace pins the weight of the drama.

Wallace tries to demonstrate the logical acrobatics, or flat out denial, of various scientists who inevitably ran afoul of Aristotelian dogma regarding limits and the limiting process. Take, for instance, Zeno's *dichotomy* [Aristotle,*Physics*, Book VI, Ch.9] which claims that a distance can never be covered because it first must be bisected, and before the midpoint can be reached that half must also be bisected, and so on. Wallace supposes that someone with a little mathematical machinery might answer this question by observing that, if the steps were chosen instead to be a geometric series, then there is no problem. This is what he calls an "impoverished" view of Zeno's paradox that is exacerbated by too much mathematical training: "Wierdly, the more standard classroom math you've had, the harder it's going to be to avoid answering in an impoverished way." Except that Wallace's argument, attributed to the mathematically inclined, is *irrelevant* not impoverished. Most mathematicians would note that Zeno's dichotomy is simply a representation of infinitely many points in a finite interval.

Regardless of whether or not Wallace was applying his argument to mathematicians in general, or to people with technical guns and no aim, the charge of philosophical poverty is worth noting because it touches on the complexity of the relationship between mathematics and popular culture; it also provides some insight into the success and failure of Wallace's attempt to bridge the gap.

Scientists and mathematicians rely on two principal defenses of their disciplines to a general audience, the first being utility and the second being beauty. The utility defense funds research, though ironically, this view falls into the impoverished category. The beauty defense aims at a broader relevance to the human experience and a deeper philosophical function. Wallace gets these two confused at times, though they are certainly not mutually exclusive.

Ignore the Grecian urn's "Beauty is truth, truth beauty," battle cry for a moment to consider the notion that the impoverished view is all that science and mathematics has to offer. In other words, everything that can be proved is impoverished in the sense that its value does not extend beyond the discipline and it's practical application. At the same time, these disciplines continue to harness fuzzy notions about the geometry and age of the universe and, of all things, infinity itself! One might reasonably extrapolate this trend to suppose that the impoverished view is all there is. This is a rather bleak conclusion since these fuzzy notions provide a lot of inspiration for all that is the human experience. Indeed, there is an ancient and still vibrant friction between the humanistic view of the world and that of science. From Berowne's dismissal of "continual plodders" who "give a name to every fixed star / [yet] have no more profit of their shining nights / than those that walk and wot not what they are." [*Love's Labors Lost*, I.1], to Walt Whitman becoming "tired and sick" by "the proofs, the figures... the charts and the diagrams," of the "learn'd astronomer", preferring to look up "in perfect silence at the stars" than to sit through such a boring lecture ["When I Heard the Learn'd Astronomer", *Leaves of Grass*(1900)], to the tension between "Bones" McCoy and Mr. Spock – the antipathy to science and abstraction that our cultural icons articulate strikes a very populist

chord.

In contrast to these, Wallace's E&M is an impassioned celebration of the science of abstraction, both its utility and beauty. At the same time, his praise of the intricacies of mathematics rings a bit like those that "wot not what they are." Wallace's contra-organizational style (imagine mathematics for MTV2) is at odds with the whole point and power of mathematics as a tool for sorting through complexity. Jim Holt, in his review for *The New Yorker* (Nov. 3, 2003), has suggested that Wallace is spoofing technical writing. I rather think that the style is, first of all, Wallace's schtick, but also a smoke screen for his limitations – he doesn't have enough command of the subject to construct a proper malaproposition. While he extols the beauty of mathematics, all the reader can fairly grasp is a frenzied interpretive dance about something to do with numbers and trees and branches and a long line of impressive people. We might as well be out with Whitman looking at these stars in perfect silence. What's refreshing is that this time the spotlight is on the proofs and figures and charts and diagrams, but, unlike other popularizations, with a fondness for contemporary sensibilities and their limitations that would keep Whitman in the lecture hall.

E&M has provoked a bit of fretting from other expert critics on the sloppy, sometimes incorrect, representation of the mathematics (see, in addition to Jim Holt's review, Michael Harris in the *Notices of the AMS* (June/July 2004) and Rudy Rucker in *Science* (Jan. 16, 2004)). Without question, the writer and his publisher should have consulted an active mathematician to parse the fallacies. It is our responsibility to be concerned with the representation of mathematics and the portrayal of mathematicians. On the other hand, it's a good idea not to interrupt while you are being praised. It demonstrates a healthy vitality of the cultural relevance of mathematics when an artist takes enough interest and has the courage to trample our reserve and concern for detail in an effort to serve up a little of what sustains us to more popular tastes.

E&M is not going to make or break Wallace's literary legacy, and he makes it clear that he knows this by frequently referring to the volume as a "booklet". If he doesn't take it seriously, then why should we? For the nonmathematician interested in an accessible and entertaining portrait of what mathematics is about, E&M isn't a bad option, despite the inaccuracies. The broad strokes are reasonably solid, and it's valuable for a general audience to see how such a seemingly mystical notion evolves to take form as a concrete mathematical object. There are, however, better, more faithful books available on the topic. For SIREV readers, honestly, I think there will be very few with the patience to tolerate what one senior colleague called Wallace's "riffing". However, it is valuable to see how mathematicians and mathematics are viewed by nonexperts because it helps us better to communicate with the broader culture. In that respect, Wallace's eyes and voice are invaluable since he is not only a prominent observer but a shaper of contemporary culture. I recommend it with the hope that you will expand your hearts to embrace the discomfort it will cause.

Emergency Glossary

fissiparous (fis·sip'·a·rous) ADJECTIVE: 1. Reproducing by biological fission. 2. Tending to break up into parts or break away from a main body; factious. [*The American Heritage Dictionary of the English Language: Fourth Edition* (2000)].

Acknowledgements

Many thanks to Haven Luke for helpful and enjoyable conversations and his careful parsing of the fallacies.