

A Convergence Proof of Local Defect Correction for Convection–Diffusion Problems

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Examples of partial differential equations with solutions that are rapidly varying functions of the spatial or temporal coordinates appear e.g. in combustion, shock hydrodynamics or transport in porous media. For boundary value problems with solutions that have one or a few small regions with high activity, a fine grid is needed in regions with high activity, whereas a coarser grid would suffice in the rest of the domain. Rather than using a truly nonuniform grid, we study a method called Local Defect Correction (LDC) that is based on local uniform grid refinement.

Several properties hold for the LDC fixed point iteration for convection-diffusion equations in two dimensions. We study the convergence behavior of the LDC method as an iterative process and derive an upper bound for the norm of the iteration matrix for the linear two-dimensional convection-diffusion equation with constant coefficients on the unit square. The research results can be extended to the case of non-constant coefficients.