A Schwarz method for a convection-diffusion problem with a corner singularity.

Deirdre Branley\textsuperscript{1}, Alan F. Hegarty\textsuperscript{1}, Helen Purtill\textsuperscript{1} and Grigory I. Shishkin\textsuperscript{2}.

\textsuperscript{1} Department of Mathematics and Statistics, University of Limerick, Plassey, Limerick, Ireland. deirdre.branley@ul.ie, alan.hegarty@ul.ie, helen.purtill@ul.ie,

\textsuperscript{2} Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences 16 S. Kovalevskaya Street, Ekaterinburg 620219, Russia shishkin@imm.uran.ru

We are concerned with two dimensional steady state convection-diffusion problems with singular outflow boundary conditions. It is well known that, where the boundary conditions are sufficiently smooth and compatible, such problems can be solved with uniform accuracy with respect to the small parameter $\varepsilon$ using a standard finite difference operator on special piece-wise uniform meshes [1, 2]. Where the outflow boundary data are only weakly regular and compatible, parameter-uniform solutions may also be obtained by this method [2]. However, orders of convergence are relatively small and pointwise errors relatively large in this case.

Numerical methods for singularly perturbed problems comprising domain decomposition and Schwarz iterative technique have been examined by a number of authors, for example in [1], [3], [4] and [5]. In particular, MacMullen et al. [5] constructed a parameter-uniform Schwarz method for singularly perturbed linear convection-diffusion problems in two dimensions with sufficiently smooth and compatible boundary data. We examine experimentally the performance of such methods extended to the class of singularly perturbed convection-diffusion problems with more general boundary conditions described below.

We consider problems of the form

$$Lu \equiv \varepsilon \Delta u_{\varepsilon} + a(x,y)\nabla u = f$$

in a domain $\Omega$, the unit square, with Dirichlet boundary conditions, where all components of $a$ are strictly positive. Such problems exhibit regular layers along the outflow boundaries, as well as a corner boundary layer at the outflow boundary corner. We deal with outflow boundary conditions, where the first derivatives are not compatible at the outflow boundary corner. We implement domain decomposition methods to isolate the neighbourhood of the singularity, along with a Schwarz iterative technique, with the aim of developing a Schwarz method to produce parameter-uniformly accurate solutions on the whole domain in the presence of such a singularity.

References


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