Layer-adapted meshes for time-dependent reaction-diffusion

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ABSTRACT

We consider singularly perturbed reaction-diffusion problems of the type

$$u_t + \mathcal{L}_\varepsilon u = f \qquad \text{in} \quad (0,1) \times (0,T],$$

where $\mathcal{L}_\varepsilon v := -\varepsilon^2 v_{xx} + rv$, subject to boundary conditions

$$u(0,t) = \gamma_0(t),\ u(1,t) = \gamma_1(t) \quad \text{in} \quad (0,T]$$

and initial condition

$$u(\cdot,0) = u^0 \qquad \text{in} \quad (0,1)$$

where $0 < \varepsilon \ll 1$, $r(x) > \rho^2 > 0$ for $x \in [0,1]$. The nature of the differential equation changes when $\varepsilon \to 0$ giving rise to boundary layers that require special attention in the design of numerical methods, in particular local refinement of the meshes used.

We study an inverse monotone difference scheme on arbitrary meshes. An maximum-norm error bound is derived that allows easy classification of various layer-adapted meshes proposed in the literature.

For example, this general result implies

$$\|u - U\|_\infty \le C \begin{cases} \tau + N^{-2} \ln^2 N & \text{for Shishkin meshes,} \\ \tau + N^{-2} & \text{for Bakhvalov meshes,} \end{cases}$$

where τ is the maximal time-step size and N the number of mesh points used in the space discretization.