Layer Capturing via Anisotropic Mesh Adaption

Simona Perotto

MOX - Dipartimento di Matematica "F. Brioschi"
Politecnico di Milano, Via Bonardi 9
I-20133 Milano, Italy
simona.perotto@mate.polimi.it

The numerical approximation of many problems in Computational Fluid Dynamics (CFD) leads often to deal with solutions exhibiting directional features, namely great variations along certain directions and less significant changes along the other ones. This is the case of boundary and internal layers typical, for instance, of the advection-diffusion and Navier-Stokes equations as well as of shocks in the case of Euler equations. In view of an efficient numerical approach to this kind of problems, it turns out advisable to resort to a suitably oriented and deformed (i.e., anisotropic) computational mesh, matching the local directional features of the solution. For a fixed solution accuracy, a considerable reduction of the number of the degrees of freedom is shown in the presence of an anisotropic grid, besides a sharper capture of the solution features.

In this regard, we have introduced a theoretically sound anisotropic framework moving from the spectral properties of the standard affine map between the reference and the general mesh element [2]. As first step, we have derived suitable anisotropic interpolation error estimates for piecewise linear finite elements. Then, in view of an anisotropic mesh adaption driven by an a posteriori error estimator, these anisotropic estimates have been merged with the standard dual-based approach proposed by R. Becker and R. Rannacher in [1]. Thus the final outcome of our analysis consists of an automatic tool able to properly orient and stretch the mesh elements so that a goal-functional of the exact solution, representing a quantity of interest, is approximated within a user-defined tolerance [3, 4].

In this communication the leading ideas of our anisotropic framework are presented together with some numerical results associated with the goal-oriented a posteriori analysis.

References

[1] R. Becker and R. Rannacher, "A feed-back approach to error control in finite element methods: basic analysis and examples", *East-West J. Numer. Math.*, **4**(4), 237–264 (1996).

[2] L. Formaggia and S. Perotto, "New anisotropic a priori error estimates", *Numer. Math.*, **8**9, 641–667 (2001).

[3] L. Formaggia and S. Perotto, "Anisotropic error estimates for elliptic problems", *Numer. Math.*, **9**4, 67–92 (2003).

[4] L. Formaggia, S. Micheletti and S. Perotto, "Anisotropic mesh adaptation in Computational Fluid Dynamics: application to the advection-diffusion-reaction and the Stokes problems.", *Appl. Numer. Math.*, **5**1 (4), 511–533 (2004).