An Adaptive Grid Method for the Solar Coronal Loop Model

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Many interesting phenomena in plasma fluid dynamics can be described within the framework of magnetohydrodynamics. Numerical studies in plasma flows frequently involve simulations with highly varying spatial and temporal scales. As a consequence, numerical methods on uniform grids are inefficient to be used, since (too) many grid points are needed to resolve the spatial structures, such as boundary and internal layers, shocks, discontinuities, shear layers, or current sheets. For the efficient study of these phenomena, adaptive grid methods are needed which automatically track and spatially resolve one or more of these structures. An interesting application within this framework can be found in and around our sun. There are several cases for which we can expect steep boundary and internal layers (see figure 1).

The first one deals with the expulsion of the magnetic flux by eddies in a solar magnetic field model. This model addresses the role of the magnetic field in a convecting plasma and the distortion of the field by cellular convection patterns for various (small) values of the resistivity (magnetic diffusion coefficient). This situation is of importance around convection cells just below the solar photosphere. Steep boundary and internal layers are formed when the magnetic induction reaches a steady-state configuration.

In this presentation we discuss another phenomenon that takes place a little bit farther from the solar interior, viz., in the solar corona. It is known that the temperature gradually decreases from the center of the sun down to values of around 10^4 degrees Kelvin at the foot of the corona (see figure 2). From that point, however, it surprisingly increases dramatically again up to several millions of degrees Kelvin forming a non-trivial transition zone (boundary layer) between the photosphere and the chromosphere. Moreover, because of the extreme temperatures, the solar corona is highly structured with closed magnetic structures which are generally known as coronal loops. It can be derived that the temperature T and pressure distribution P in the loop as a function of a mass-coordinate z satisfy the following PDE model:

$$\frac{5}{2}\frac{P}{T}\frac{\partial T}{\partial t} - \frac{\partial P}{\partial t} = \epsilon\ \frac{P}{T}\frac{\partial}{\partial z}(T^{\frac{3}{2}}P\frac{\partial T}{\partial z}) + E_H - P^2\chi(T), \tag{1}$$

where $P(z,t) = P_0(t) - \mu z$, E_H is a heating function, $\chi(T)$ the radiative loss function and ϵ a small parameter representing the thermal conductivity in the loop. Near the base of the loop there are two adjacent boundary layers where the temperature increases very quickly when moving upward in the loop; in these thin layers the pressure in nearly constant. We will examine the nature of this special boundary layer via the theory of significant degenerations and also in terms of a dynamical system of the steady-state of PDE (1) in which a non-trivial saddle-center connection occurs. A complicating factor is the fact that we also need to take into account the so-called loop-condition:

$$\mathcal{L} = 2M\mathcal{R}\int_0^1 \frac{T}{P}\,dz = \ constant, \tag{2}$$

with gasconstant $\mathcal{R}$, total mass in the loop M and (half) looplength $\mathcal{L}$.

To support and confirm the theory, we have applied an adaptive grid technique, based on an equidistribution principle with additional smoothing properties, to numerically simulate the forming of the thin boundary layer.

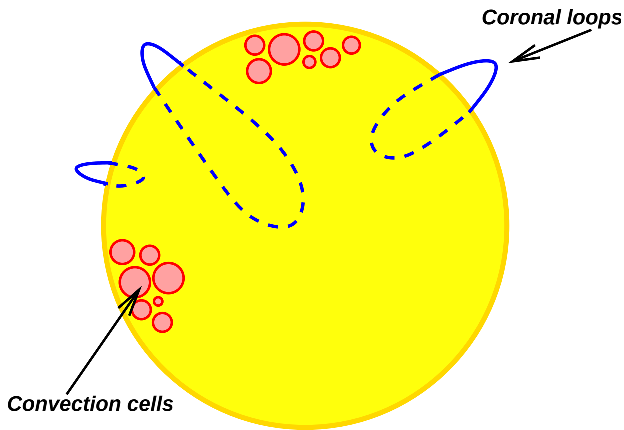

Figure 1: Convection cells and coronal loops in, and on top of, the sun in the form of magnetic field lines.

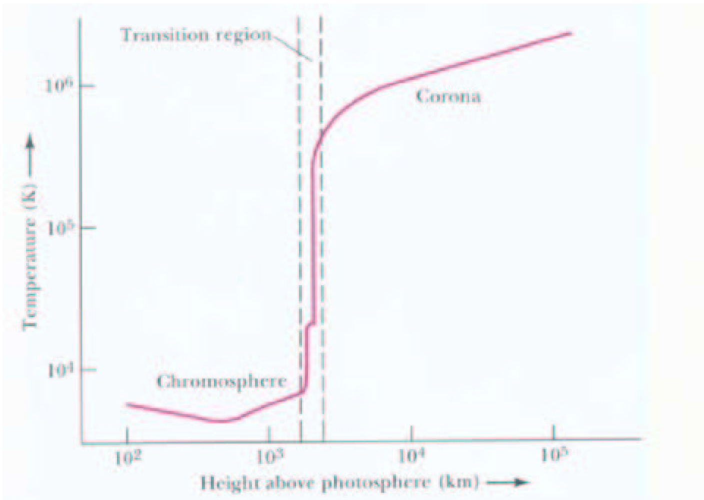

Figure 2: The temperature distribution and the steep transition zone in the outer layers of the sun.

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