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# Boundary Layer Interaction with External Disturbances

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Paper presents results of theoretical and experimental investigations of the interaction process of three basic disturbance modes: acoustic, vortical and thermal, with the supersonic boundary layer (BL). Excitation of Tollmien-Schlichting (TS) waves in the supersonic BL by a pair of acoustic waves have been also studied. Receptivity coefficients and the range of wave parameters where generation of TS waves takes place are determined numerically. Problem for the disturbances of compressible BL caused by a small amplitude sinous surface roughness has been formulated mathematically and solved in the linear approximation. Nonlinear excitation of the instability waves by the sound wave and the surface roughness was investigated . It is established, that the maximal excitation occurs when angle between the wave front of roughness and the mean flow direction is equal to the Mach angle. Paper present results of detailed investigation of the interaction of vortical and thermal perturbations, generated by a turbulizer grid, with the supersonic BL.

## Nomenclature

| | | |
|---|---|---|
| $A_{max}$ | = | mass flux perturbation maximum |
| $R$ | = | Reynolds number |
| Pr | = | Prandtl number |
| $M$ | = | Mach number |
| $\chi$ | = | angle between the wave front and the plate leading edge |
| $c$ | = | phase speed |
| $\alpha,\beta$ | = | streamwise and spanwise wave numbers |
| $\omega$ | = | frequency |
| $m$ * | = | mass flux relative amplitude |
| $K$ | = | receptivity coefficient |
| $U_{max}$ | = | streamwise velocity disturbance maximum across the BL |
| $I$ | = | growth rate |

## 1. Introduction

Generation of initial unstable eigen disturbances inside the BL by external perturbations is an actual problem nowadays in the investigation of laminar-turbulent transition. Morkovin [1] was the first who has formulated this so called BL receptivity problem. Up to now a lot of experimental and theoretical works studying subsonic BL has been performed. The detailed review of these efforts can be found in [2,3]. The knowledge about the supersonic BL is much shorter at the present moment. The existing papers are devoted mainly to the investigation of external acoustic field interaction with a supersonic BL on a smooth flat plate [4-6]. However an oncoming supersonic flow always comprises not only acoustic but also vortical and thermal (entropy) perturbations [7]. To the authors knowledge, there is a unique paper [8], where the interaction of such hydrodynamic waves with the supersonic BL has been considered. This paper presents results concerning irrotational external disturbances with zero damping in the direction of the main flow. Present paper is devoted to the investigation of disturbance excitation in sub- and supersonic BL by external vortical and thermal waves. Interaction of the acoustic waves with the BL on the non-smooth surface has also been investigated and is reported here.

## 2. Basic equations

**Parallel approximation.** In the parallel approximation flow parameters do not depend of the streamwise coordinate. We normalize the basic equations in a normal way, introducing characteristic length (Blasius scale) $\delta = \sqrt{x^* \mu_e / \rho_e U_e}$, time scale $t = \delta / U_e$, where $x^*$ – is the distance from the plate leading edge. Consider BL on a flat insulated plate. We introduce nondimensional viscosity, velocity and temperature by their values at BL outer edge: $\mu_e, U_e, T_e$. To describe the disturbance flow-field we use linear approximation which is applicable for infinitesimal amplitudes of incident acoustic wave. Parameters of an external wave could be described by means of the vector $\vec{Q}_0(x_1, y, z_1, t) = \varepsilon \vec{q}_0 \exp[i(\alpha_1 x_1 + \beta_1 z_1 + \lambda_0 y - \omega_1 t)]$, where $\varepsilon$ – is wave amplitude in the accepted normalization. Here the coordinate $x_1$ is directed parallel to the main flow, $y$ – is normal to the plate, $z_1$ – in spanwise direction. Perturbations inside the BL could be expressed by a relation $\vec{Q} = \vec{q}(y) \exp[i(\alpha x_1 + \beta z_1 - \omega t)]$. Then the eight order system of governing equations under some additional assumptions could be reduced to the six-order system [1], the well-known Dunn-Lin system [8]:

$$\frac{dq_i}{dy} = a_{ij} q_j, \quad i, j = 1, \ldots, 6. \tag{1}$$

Here $q_1$ and $\alpha q_3$ – are disturbance velocities in $x$ – and $y$ – directions, $\gamma (\mathrm{M} \cos\chi)^2 q_4$ and $q_5$ – are disturbance pressure and temperature; $q_2$ and $q_6$ – are the derivatives of $q_1$ and $q_5$ in $y$, $\gamma$ – specific heat ratio, $\mathrm{M} = U_e / a$ – is flow Mach number, $a$ – is speed of sound. Coefficients $a_{ij}$ do not depend of $y$, wave parameters $\bar{\alpha} = \sqrt{\alpha^2 + \beta^2}$ and $\omega = \alpha c$, the Reynolds number $\bar{R} = R \cos\chi$ and the Mach number $\bar{\mathrm{M}} = \mathrm{M} \cos\chi$, where $R = U_e \delta / \nu_e$, $\chi = \mathrm{arctg}(\beta/\alpha)$. The ordinary no-slip boundary conditions are applied on the surface

$$q_1(0) = q_3(0) = q_5(0) = 0. \tag{2}$$

Outside of the BL the solution could be expressed as

$$\vec{q} = \vec{q}^{\,(0)} e^{i\lambda_0 y} + I_1 \vec{q}^{\,(1)} e^{-i\lambda_1 y} + I_2 \vec{q}^{\,(2)} e^{-\lambda_2 y} + I_3 \vec{q}^{\,(3)} e^{-\lambda_3 y}, \tag{3}$$

where $\lambda_1 \approx \bar{\alpha} \sqrt{\bar{\mathrm{M}}^2 (1 - c)^2 - 1}$, $\lambda_2 \approx \sqrt{i \bar{\alpha} \bar{R} (1 - c) + \bar{\alpha}^2}$, $\lambda_3 = \sqrt{\mathrm{Pr}\, i \bar{\alpha} \bar{R} (1 - c) + \bar{\alpha}^2}$, $\lambda_0 = -\lambda_k$, Pr – is the Prandtl number. The first term in (3) corresponds to external wave under the accepted normalization. The third and fourth terms in (3) describe thermal and vorticity perturbations. The second term describes the potential (acoustic) wave radiated from the BL. One have to consider the wave corresponding to $\lambda_k$, as reflected from the BL. System (1–3) allows calculate the reflection coefficient $I_k$ of the wave as an eigen-value of the problem, while distribution of the disturbance amplitude inside the BL $q_i(y)$ could be determined as the eigen-function of the correspondent boundary value problem.

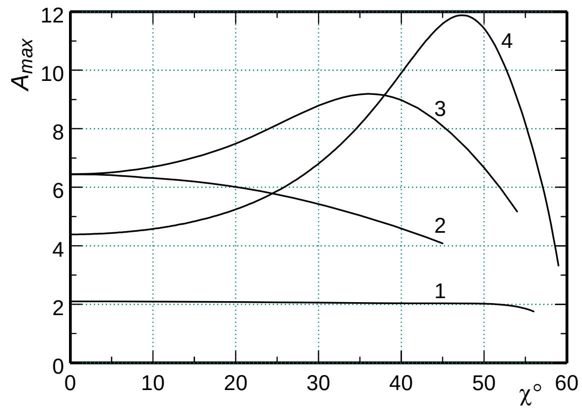


**Fig.1**

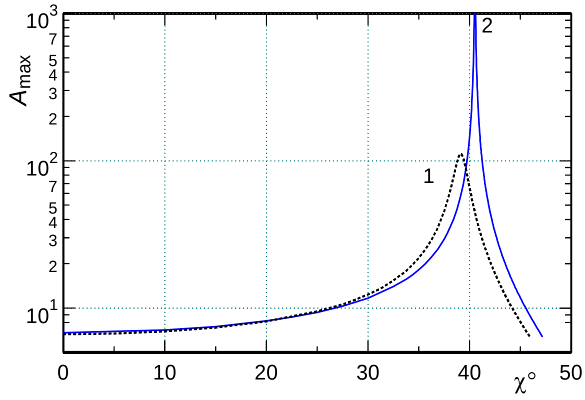


**Fig.2**

found in references [10,13].

For three-wave resonant interaction problem one can obtain parabolized system of equations like: $\mathbf{K}\hat{\mathbf{Z}} = \hat{\mathbf{N}}(\hat{\mathbf{Z}}_1 + \hat{\mathbf{Z}}_2)$, where $\mathbf{K}$, $\hat{\mathbf{N}}$ – are linear and nonlinear differential operators, $\hat{\mathbf{Z}}_1, \hat{\mathbf{Z}}_2$ – are solutions of corresponding linear equations, and operator $\mathbf{K} \equiv \partial/\partial\eta = A + D\,\partial/\partial R$. The structure of the nonlinear operator can be found in [14].

## 3. Interaction of acoustic waves with a boundary layer.

**Steady Mach waves and streamwise acoustic waves.** We have performed computations for different Mach and Reynolds numbers as well as for different frequencies and wave vector orientation angles of the incident acoustic wave relative to mean flow direction. The viscosity-temperature dependency was assumed in the shape of Sutherland law. Constant Prandtl number $\Pr = 0.72$ was chosen.

First we present here the results for the case of BL interaction with steady ($\chi = \omega_1 = 0$) potential periodic in streamwise and spanwise directions external perturbations. Such stationary inhomogeneities of the external flow could be produced for example, by roughness on supersonic wind tunnel test section walls. These perturbations are also the Mach waves of zero frequency, capable to induce early transition in the model BL. Fig.1 presents computational results for $A_{\max} = A_{\max}(\chi)$ and parameters: $M = 2$, $R = 500$, $\alpha$ =0.2(1), 0.05(2), 0.03(3), 0.02(4)). One can easily see that for short-wave external perturbation ($\alpha \geq 0.05$) interaction is stronger for two-dimensional (2D) waves (2D, $\chi = 0^{\circ}$), while for $\alpha \leq 0.03$ 3D disturbance induces inside the BL inhomogeneity with higher amplitude than at $\chi \neq 0^{\circ}$.

For streamwise acoustic wave (SAW) ($\lambda_1 = 0$, $\chi = 1 - 1/\bar{M}$) the separation onto the incident and reflected waves makes no sense, and therefore the indefiniteness in mathematical formulation of the problem (1–3) arises. Acoustic wave propagating along the model surface induces on the BL outer edge an additional flow with non-zero values of $\theta_3(\delta_1)$. Functions $A_{\max}(\chi)$ are shown at Fig.2. The obtained results are similar to the case of steady inhomogeneities (compare with Fig.1), however maximal values of $A$ are much higher ($A_{\max} > 10^2$ at $\chi \approx 40^{\circ}$). Theoretical results for acoustic waves of non-zero incidence angle were presented earlier in [12].

**Diffraction of acoustic waves on plate leading edge.** Experiments [6] dealt with the interaction of the BL only with the diffraction wave. At large distances from plate leading edge the diffraction wave at BL outer edge becomes SAW with wave fronts inclined under different angles relative to the plate leading edge. Theory predicts that SAW is able to induce oscillations of high intensity inside the BL. Therefore small amplitude diffraction waves induce strong fluctuations inside the BL. This fact explains a strong influence of small-amplitude acoustic field acting in the vicinity of the leading edge onto the BL eigen unstable oscillations far downstream. Unfortunately it is not possible to measure the amplitude of the diffraction wave in the experiment because it is very small. This amplitude, therefore, should be assumed to be given for theoretical analysis and as a consequence it is not possible to make a direct comparison of theory with measurements. Therefore we have developed a procedure to calculate the intensity of the diffraction wave from the known (for instance, measured) distribution of the mass flux oscillations in the sound field at the level of the plate upstream of it.

In the experiment we have detected mass flux perturbations at a given frequency in the region $x < 0$ at $y = 0$. The acquired data were later processed by means of the Fourier transformation in time $t$ and spanwise coordinate $z$. Thus from measurement one can obtain the distribution $\bar{m}_1(\omega,\beta,x,y=0)$ at $x < 0$, where $\omega$ – is reduced frequency, $\beta$ – is the wave number in $z$-direction. Mass flux produced by the acoustic wave is $m_1(t,z,x,y) = \rho +$, where $\rho$, $u$ are perturbations of density and streamwise velocity, which should satisfy to the wave equation. Using Fourier transformation and the equations of gas dynamics, one can show, that mass flux fluctuations along the plate could be completely determined by means of the jump of normal-to-the-wall disturbance velocity at the plate leading edge. This normal disturbance velocity $\bar{v}(x)$ upstream of the plate leading edge is determined by the distribution of the mass flux in the region $x < 0$ by the following relation: $\bar{v}^0(0) = -\frac{1}{M^2-1}\frac{1}{2}\alpha\sqrt{M^2-1}\,\tilde{m}(\alpha_0)$. Mass flux at the plate surface as a function of $x$ has the form: $\bar{m}_1(x,\beta,\omega) = -\frac{1}{2}\alpha\sqrt{M^2-1}\,\tilde{m}(\alpha_0)J_0(\alpha x)\exp(i\alpha_0 x)$. Here $J_0$ – is the Bessel function of zero order, $\alpha_0 = M^2\omega/(M^2-1)$, $\alpha^2 = \alpha_0^2 + \beta^2/(M^2-1)$, $\alpha_0^2 = M^2\omega^2/(M^2-1)^2$. At large values of $x$ mass flux oscillations could be described by the following identity: $\bar{m}_1(x,\beta,\omega) \approx -\frac{\sqrt{M^2-1}}{2}\tilde{m}(\alpha_0)\sqrt{\frac{\alpha}{2\pi x}} \times \{\exp[i(\alpha_1 x - \pi/4)] + \exp[i(\alpha_2 x + \pi/4)]\}$, where $\alpha_{1,2} = \omega\bar{M}/(\bar{M} \mp 1)$, $\bar{M} = M\cos\chi$, $\chi = \arctan(\beta/\alpha_{1,2})$. That means that intensity of the mass flux fluctuation is reducing with distance from the leading edge and is dependent of the orientation of the incident wave. The obtained formulas give the relations of the mass flux in the region $x > 0$ with its distribution at $x < 0$, that solves the problem fromulated in the beginning. Unfortunately theoretical results obtained here have not been compared with measurements up to now. Therefore it is difficult to confirm their reliability.

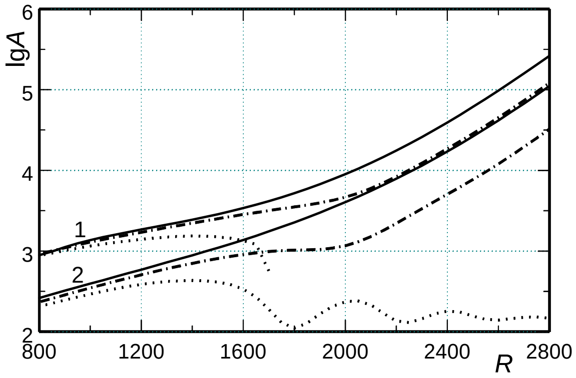


**Fig.3**

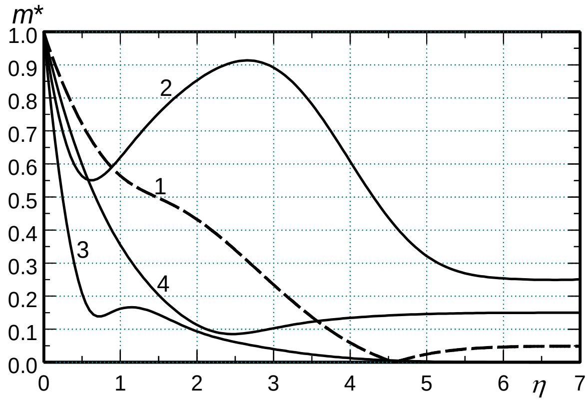


**Fig.4**

**Excitation of instability waves in the BL by a pair of acoustic waves.** Fig.3 demonstrates the process of excitation of Tollmien-Schlichting wave by a pair of sound waves, one of them is steady and another one is at the frequency $w = w_1 = 4\times10^{-6}$ with wave number $b = b_1 + b_2 = 2\times10^{-5}$ at $M = 2$. Curve 1 corresponds to $a_1 = 3{,}1\times10^{-5}$ and $b_1 = 3.7\times10^{-5}$ for different values of $a_2$ of steady waves: solid lines are for $a_2 = -2{,}1\times10^{-5}$, while dash-dotted lines represent $a_2 = -1{,}9\times10^{-5}$, and dotted are for $a_2 = -1{,}8\times10^{-5}$. For $R$ being high enough in the region $a_2 = -1{,}9\times10^{-5}$ , $-2{,}1\times10^{-5}$ the excited oscillations do not actually differ from eigen unstable disturbance. Outside of this region Tollmien-Schlichting wave is not excited. Similar results obtained at $a_1 = 1{,}01\times10^{-6}$ and $b_1 = 5{,}73\times10^{-6}$, are presented by curves 2. Solid lines represent the value $a_2 = 9{,}5\times10^{-6}$, dash-dotted lines are for $a_2 = 1{,}1\times10^{-5}$, dotted - $a_2 = 1{,}2\times10^{-5}$.

**Acoustic excitation of instability waves in the BL on the plate with roughness.** Curves 1-3 at Fig.4 show profiles of the relative amplitudes of the mass flux $m^* = \left|\tilde{m}/\tilde{m}_w\right|$, $\tilde{m} = \rho\tilde{u} + u\tilde{\rho}$, computed for $R = 1000$; $\alpha = 3\cdot10^{-5}$ (1 — $M = 0.02$; $\chi = 0$; 2 — $M = 2$; $\chi = 0$; 3 — $M = 1$; $\chi = 0$; 4 — $M = 2$; $\chi = 60^0$). Subscript $w$ stands here for values at the wall. One can easily see that at $M = 1$ (ß=0) mass flux disturbances in the main part of the BL are in reality zero. Considerable damping of disturbances takes also place for oblique roughness waves at $\bar{M} = 1$ ( $\bar{M} = M\cos\chi$, $\chi = \mathrm{arctg}\,(\beta/\alpha)$), (curve 4).

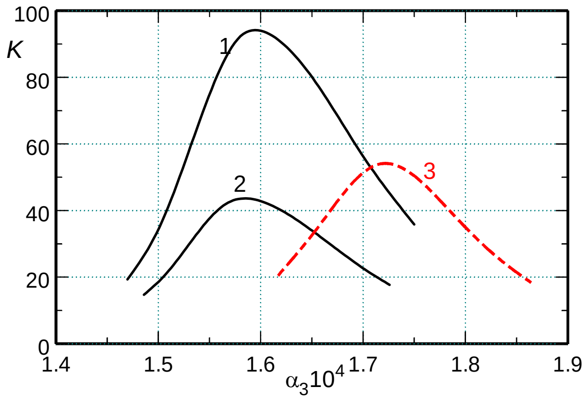


**Fig.5**

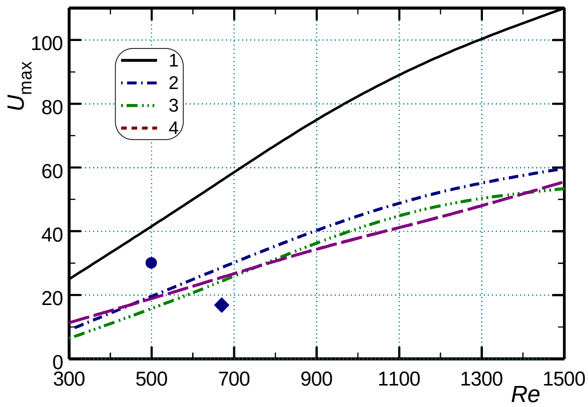


**Fig.6**

Efficiency of the excitation of unstable wave is based upon computations of the dependency of maximum across BL value of the mass flux $A$ from $R$. Growth rates $I = 0,5 d\ln A/dR$, for M = 0,6; $\omega$ = 0,56·10$^{-4}$; $\alpha_1$ = –0,7·10$^{-4}$ and different values of $\alpha_2$ have been computed up to the end of the BL instability range. In the range $\alpha_2$ = (2,17 – 2.45)·10$^{-4}$ disturbances at large enough $R$ are identified as to be a Tollmien-Schlichting wave. Outside of this range the deviation becomes large, and after that the disturbances become stable. In computations the disturbance parameters were related to $\tilde{y}_w p_5$M, while their real amplitudes – to $\delta_w p_s$M. Here $d_w = |\tilde{y}_w|$ – is the amplitude of the wave of roughness, while $p_s = |p_5|$ and $p_s$M in the order $R^{-1}$ correspond to amplitudes of pressure and velocity fluctuations in the incident acoustic wave.

The parameter of the excited Tollmien-Schlichting wave which is not dependent of $R$ is $K = A/\bar{A}_{TS}$, where $\bar{A}_{TS}$ – is a computed by stability theory relative (from the beginning of growth) amplitude. $K$ defines the receptivity of the BL to external perturbations. Maximal receptivity coefficient (in the given range of $\alpha_2$) is $K$ = 94,2. In the reference system of the flow, positive values of $\theta_1$ (in our case $\theta_1$ = 67,8°) correspond to positive components of the sound wave vector that means in the direction of plate motion, i.e. such sound waves are downstream (not upstream) propagating. Greatest values of $K$ in the range of negative angles correspond to $\theta_1$ = –75,9°. Curves 1, 2 at Fig.5 represent $K$ as a function of combination wave number $\alpha_3 = \alpha_1 + \alpha_2$ for a given direction of sound propagation. In both cases maximum is reached at values of $\alpha_3$ close to Tollmien-Schlichting wave numbers in the beginning of their growth range $\alpha^*$ = 1,61·10$^{-4}$. Curve 3 presents data of paper [15] for $M = 0$. The observed shift to higher $\alpha_3$ could be explained by higher values of $\alpha^*$ = 1,73·10$^{-4}$. At $M = 0$ it makes no difference in which direction the sound wave propagates – upstream or downstream. However already at $M = 0.6$ the maximal coefficients $K$ for negative and positive values of the angle $\theta_1$ differs in more than 2 times.

Computations have also been performed for oblique roughness waviness $\beta_2$ = 4·10$^{-5}$ и 2·10$^{-5}$ at $M = 2$; $\omega$ =10$^{-5}$. Excited disturbances are practically indistinguishable from Tollmien-Schlichting waves, starting from $R \gg 2000$, and they propagate under angles 62° and 43° relative to main flow direction correspondingly. In the whole computations show that already at subsonic speed large enough the receptivity is strongly dependent on the sound propagation direction. At $M > 1$ the instability could be excited by disturbances with wavelength much smaller than Tollmien-Schlichting wavelength. Another important peculiarity of the supersonic flowing is that the maximal receptivity corresponds to the angle between the front of the roughness waviness and the direction of mean flow being equal to Mach angle.

## 4. Interaction of the vorticity perturbations with the BL.

The results obtained for external vorticity disturbances have been normalized by the velocity magnitude $u=(\tilde{v}^2+\tilde{w}^2)^{1/2}$ near the turbulizer grid. The parameters of the problem are: $\alpha_i$, R, $x_0$, where $\alpha_i$ – is external disturbance damping rate along streamwise coordinate, $x_0$ – nondimensional distance from the grid to plate leading edge, x - nondimensional distance from plate leading edge along main stream. Computations have been performed for the BL $\delta=\eta_1=6$ thick. Fig.6 presents streamwise disturbance velocity maxima $U_{\max}$ as function of the Reynolds number $R$ at $(M = 0)$. Presented data were obtained under the condition k=β/3: curve *1* – shows data of [16], curves *2,3*- results of the present investigation (in local parallel approximation and by means of PSE correspondingly). Curve *4* demonstrates our results at k=β, corresponding to maximal values $U_{\max}$ as function of $b$ . Also at this diagram the symbol • depicts measured value taken from the paper [17], while symbol ♦ shows measurements of the paper [18]. The reasons of the discrepancy of our data with [16] are not clear up to now. However under the conditions $b\,\mathrm{R} = 1 = b\,\mathrm{R}^2$ our results are in a complete agreement with theory [19], which should be applicable in this region.

**Results for supersonic flow.** The analysis shows that the dependency of $U_{\max}$ from the Reynolds number at $M > 2$ (for external vorticity perturbations) is very similar to the case of Mach number $M = 0$. The low efficiency of the streamwise velocity generation takes place in the BL in comparison to the case of subsonic velocities, that is in a good agreement with conclusions of [9]. Computations show, that $U_{\max}$ is monotonously decreasing with the Mach number. This conclusion is demonstrated at Fig.7 for $R = 600$, $-\alpha_i=10^{-6}$, $x_0= 6.4\cdot 10^5$ .

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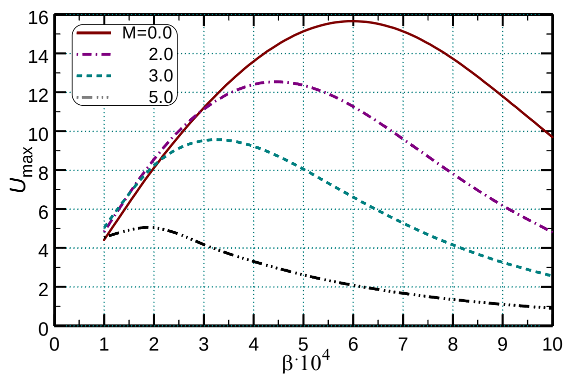


**Fig.7**

## References

[1] M.V. Morkovin, "Critical evaluation of transition from laminar to turbulent shear layer with emphasis on hypersonically traveling bodies", Tech.Rep. AFFDL №68-149 (1969).

[2] Yu.S. Kachanov, "Physical mechanisms of laminar-boundary-layer transition", *Annu.Rev.Fluid Mech*., **26**, 411 (1994).

[3] A.V. Boyko, G.R. Greek, A.V. Dovgal, V.V. Kozlov, *Turbulence origin in near-wall flows*. Novosibirsk: Science (1999) (in Russian).

[4] S.A. Gaponov, "On the interaction of a supersonic boundary layer with acoustic disturbances", *Thermophysics and Aeromechanics*, **2**, №3, 209-217 (1993) (in Russian).

[5] A.V. Fedorov, A.P. Hohlov, "Excitation of unstable modes in a supersonic boundary layer by acoustic waves", *Fluid Dynamics*, №4, 67-71 (1991) (in Russian).

[6] N.V. Semionov, A.D. Kosinov, A.A. Maslov, "Experimental investigation of supersonic boundary layer receptivity", *Transitional boundary layers in aeronautics,* edited by R.A.W.M.Henkes and J.L. van Ingen. North-Holland, Amsterdam, 413-420 (1996).

[7] L.S.G. Kovasznay, "Turbulence in supersonic flow", *J.Aeron.Sci*., **20**, №10, 1953, pp.657-674.

[8] K.J.A. Westin, A.A. Bakchinov, V.V. Kozlov, P.H. Alfredsson, "Experiments on localized disturbances in a flat plate boundary layer. Pt 1: The receptivity and evolution of a localized free stream disturbances", *Eur. J. Mech. B. Fluids*, **17**, №6, 823-846 (1998).

[9] S.A. Gaponov, A.V. Yudin, "Interaction of hydrodynamic external perturbations with a boundary layer", *Appl.Mech.Techn.Phys.*, **43**, №1, 100-107 (2002) (in Russian).

[10] G.V. Petrov, "New parabolized stability equations for compressible boundary layer", *Appl. Mech.Techn.Phys.*, **41**, №1, 63-69 (2000) (in Russian).

[11] S.A. Gaponov, A.A. Maslov, *Disturbance development in compressible flows*. Novosibirsk: Science (1980) (in Russian).

[12] S.A. Gaponov "On the interaction of a supersonic boundary layer with acoustic waves", *Thermophysics and aeromechanics*, №3, 181-189 (1995) (in Russian).

[13] G.V. Petrov, "Reaction of a supersonic boundary layer onto acoustic influence", *Thermophysics and aeromechanics*, **8**, №1, 77-86 (2001) (in Russian).

[14] G.V. Petrov, "Generation of the Tolmien-Schlichting wave in a supersonic boundary layer by two harmonic acoustic waves", *Appl.Mech.Techn.Phys.*, **43**, №1, 76-82 (2002) (in Russian).

[15] J.D**.** Crouch, *Theoretical studies on the receptivity of boundary layers*. AIAA Paper №94-2224 (1994).

[16] F.P. Bertolotti, "Response of the Blasius boundary layer to free-stream vorticity", *Physics Fluids*, **9**, №8, 2286-2299 (1997).

[17] K.J. Westin, A.V. Boiko, V.V. Kozlov, B. Klingmann, P.H. Alfredsson, "Experiments in a boundary layer subjected to free stream turbulence. Part 1. Boundary layer structure and receptivity", *J.Fluid Mech*., **281**, 193-218 (1994).

[18] M. Matsubara, P.H. Alfredsson, "Disturbance growth in boundary layer subjected to free-stream turbulence", *J.Fluid Mech*., **430**, 149-168 (2001).

[19] A. Tumin, "A model of spatial algebraic growth in a boundary layer subjected to a streamwise pressure gradient", *Physics Fluids*, **13**, №5, 1521-1523 (2001).