Asymptotic analysis of the mixed convection flow past a horizontal plate near the trailing edge

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1. Introduction

The flow near the trailing edge of a horizontal heated plate which is aligned under a small angle of attack ϕ to the oncoming parallel flow with velocity U_∞ in the limit of large Reynolds Re and large Grashof $\mathrm{Gr} = g\beta\Delta T L^3/\nu^2$ number will be investigated (see figure 1). As usual β and ν denote the isobaric expansion coefficient and the kinematic viscosity, respectively. The difference between the plate temperature and the temperature of the oncoming fluid is ΔT and L is the length of the plate. A measure for the influence of the buoyancy onto the boundary layer flow along a horizontal plate is the buoyancy parameter $K = \mathrm{Gr}\,\mathrm{Re}^{-5/2}$ as defined in [6].

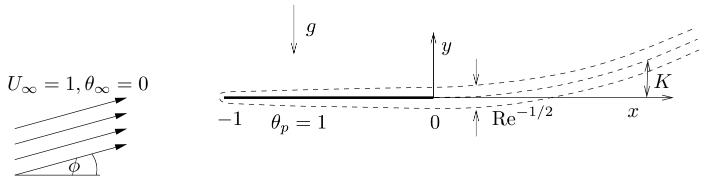

Figure 1: Mixed convection flow past a horizontal plate.

The starting point of the analysis are the Navier Stokes equations for an incompressible fluid using Bousinesq's approximation to take buoyancy forces into account and the energy equation.

$$\begin{aligned} uu_x + vu_y &= -p_x + \frac{1}{\mathrm{Re}}(u_{xx} + u_{yy}), \\ uv_x + vv_y &= -p_y + \frac{1}{\mathrm{Re}}(v_{xx} + v_{yy}) + \frac{\mathrm{Gr}}{\mathrm{Re}^2}\vartheta, \\ u\theta_x + v\theta_y &= \frac{1}{\mathrm{Re}\,\mathrm{Pr}}(\theta_{xx} + \theta_{yy}), \\ u_x + u_y&, \end{aligned} \tag{1}$$

subjected to the asymptotic boundary conditions

$$u = 1, \quad v = \phi, \quad \theta = 0 \tag{2}$$

and the boundary conditions at the plate

$$u(x,0) = v(x,0) = 0, \quad \theta(x,0) = 1, \quad - < x < 0. \tag{3}$$

Additionally to the above mentioned dimensionless parameters the Prandtl number Pr, which is assumed to be of order one, and the angle of attack ϕ enter the problem.

The global structure of the flow field is shown in figure 1. The flow around the plate is a potential flow with the exception of the boundary layer at the plate and the wake where viscous effects play a role. Near the trailing edge of the plate the boundary layer interacts (locally) with the potential flow and sub-layers according to triple deck theory ([1, 2]) will be introduced. To apply the triple deck analysis it turns out that the buoyancy parameter K and the angle of attack ϕ have to be of the order $\mathrm{Re}^{-1/4}$. Thus we define the reduced buoyancy parameter $\kappa = K\,\mathrm{Re}^{1/4}$ and the reduced inclination parameter $\lambda = \phi K \sqrt{\mathrm{Re}}$. We note that the choice of the magnitude of ϕ is not only dictated by the trailing edge analysis, but it is a consequence of the analysis of the far field (see [8]). As we will see the inclination parameter λ will play no role in the trailing edge analysis. Only for positive values of λ an outer potential flow field exists [8]. We remark that in case of symmetric flow conditions (upper side of the plate heated, lower side cooled) the inter action mechanism would allow K to be larger, namely of order $Re^{-1/8}$. After a short review of the interaction of the wake (section 2) with the potential flow (section 3) the focus of the present paper will be the analysis of the flow near the trailing edge (section 4). A numerical solution reveals that the interaction pressure is discontinuous at the trailing edge (section 5). Thus new sub-layers are introduced to resolve the discontinuity (section 6).

In this paper we will use the following notation for the variables in different layers. Consider a sub-layer of the dimensions $\mathrm{Re}^{-\alpha/8}$ in x- and $\mathrm{Re}^{-\beta/8}$ in y-direction. The corresponding independent variables are denoted by $x^{(\alpha)} = x\mathrm{Re}^{\alpha/8}$ and $y^{(\beta)} = y\mathrm{Re}^{\beta/8}$. A dependent variable defined on that sub-layer, e.g. u, will be denoted by $u^{(\alpha,\beta)} = u^{(\alpha,\beta)}(x^{(\alpha)}, y^{(\beta)})$.

2. Boundary Layer and Wake

The boundary layer and wake are of the thickness $\mathrm{Re}^{-1/2}$. Since we have to expect an inclination of the wake we introduce the stretched y-coordinate as

$$y^{(4)} = \left(y - \mathrm{Re}^{-1/4} y_w\right) \mathrm{Re}^{1/2},$$

where the center line of the wake is given by $y = \mathrm{Re}^{-1/4} y_w(x)$. In the boundary layer $-1 < x < 0$ we set $y_w(x) = 0$. Perturbations of the pressure are expected to be of the order of the buoyancy parameter. Thus the pressure can be expanded in the form

$$p(x, y) \sim \mathrm{Re}^{-1/4} p_0^{(\alpha,\beta)}(x^{(\alpha)}, y^{(\beta)}) + \ldots \quad . \tag{4}$$

We remark that in the potential flow region we have $\alpha = \beta = 0$ and in the wake and boundary layer $(\alpha, \beta) = (0, 4)$. Applying the usual scaling for the boundary layer (and wake) we obtain for the leading order terms the x-momentum and continuity equation.

$$u_0^{(0,4)} \frac{\partial u_0^{(0,4)}}{\partial x^{(0)}} + v_0^{(0,4)} \frac{\partial u_0^{(0,4)}}{\partial y^{(4)}} = \kappa y_w' \theta^{(0,4)} + \frac{\partial^2 u_0^{(0,4)}}{\partial {y^{(0)}}^2}, \qquad \frac{\partial u_0^{(0,4)}}{\partial x^{(0)}} + \frac{\partial v_0^{(0,4)}}{\partial y^{(4)}} = 0. \tag{5}$$

The $y-$momentum equation reduces to

$$p_0^{(0,4)} = \kappa \theta_0^{(0,4)}. \tag{6}$$

We remark that due to the inclination of the wake the hydrostatic pressure gradient has a non vanishing component tangential to the wake. The inclination of the wake has to be determined from the potential flow. As mentioned earlier at the plate $y_w' = 0$ holds and thus we have the to leading order the Blasius solution for the boundary layer flow

$$u \sim f_B' \left(\frac{y^{(4)}}{\sqrt{x^{(0)} + 1}} \right), \quad \theta \sim \theta_B \left(\frac{y^{(4)}}{\sqrt{x^{(0)} + 1}} \right), \quad -1 < x < 0, \tag{7}$$

where f_B is the Blasius similarity solution and θ_B is the corresponding similarity solution for the temperature profile.

3. Potential flow

Integrating (6) across the wake we obtain

$$p_0^{(0,0)}(x,0+) - p_0^{(0,0)}(x,0-) = p_0^{(0,4)}(x,+\infty) - p_0^{(0,0)}(x,-\infty) = \kappa \int_{-\infty}^{\infty} \theta^{(0,4)}(y^{(4)})\,\mathrm{d}y^{(4)}, \tag{8}$$

that across the wake there is due to the temperature perturbation in wake a hydrostatic pressure difference. Thus the potential flow has to satisfy the pressure jump condition (8), the condition the flow has to be tangential to the plate $v(x,0) = 0$ for $-1 < x < 0$ and the asymptotic boundary condition (2). Using the notation of complex functions we decompose the potential flow field as follows

$$u - iv = 1 - i\phi\sqrt{\frac{z}{z+1}} + \mathrm{Re}^{-1/4}\kappa(u_1 - iv_1) + ..., \tag{9}$$

with $z = x + iy$. The first term corresponds to the potential flow around the plate under an angle of attack ϕ. The second part is due hydrostatic pressure difference across the wake. Using (9) the inclination of the wake is given by

$$y_w'(x) = \frac{\lambda}{\kappa}\sqrt{\frac{x}{x+1}} + \kappa v_1(x,0). \tag{10}$$

For v_1 an integral equation can be derived. Its solution is given by

$$v_1(x) = \frac{1}{2\pi}\sqrt{\frac{x}{x+1}} \int_0^{\infty} \frac{\gamma_w(\xi)}{x-\xi}\sqrt{\frac{\xi+1}{\xi}}\,\mathrm{d}\xi, \tag{11}$$

(see [8]) where $\gamma_w(x) = \int_{-\infty}^{\infty} \theta_0^{(0,4)}(x, y^{(4)})\,\mathrm{d}y^{(4)}$ can be interpreted as a vortex distribution along the center line of the wake which compensates the hydrostatic pressure difference across the wake (cf. [7]). We remark that for non-vanishing values of κ the flow in the wake and the potential flow correction due to buoyancy have to be solved simultaneously. A detailed analysis and discussion of the solution can be found in [8].

4. Trailing Edge

For the analysis of the flow field near the trailing edge the velocities, pressure and temperature are decomposed into a symmetric and anti-symmetric part.

$$\bar{u}(x,y) = \frac{u(x,y) + u(x,-y)}{2}, \quad \Delta u = \frac{u(x,y) - u(x,-y)}{\mathrm{Re}^{-1/4}\kappa}. \tag{12}$$

All other dependent variables are decomposed accordingly. To leading order for the symmetric part the classical triple deck problem [1, 2] is obtained. Here we recall some properties of the interaction pressure $\bar{p}_1^{(3,5)}$ and the displacement thickness $\bar{A}$

$$\bar{A} \sim a_s \left(x^{(3)}\right)^{1/3}, \quad x^{(3)} \to \infty, \tag{13}$$

with the constant $a_s = 0.892$. In analogy to the velocity profile of the symmetric part in the main deck the temperature profile of the symmetric part is given as

$$\theta \sim \theta_B(y^{(4)}) + \mathrm{Re}^{-1/8}\bar{A}(x^{(3)})\theta_B'(y^{(4)}). \tag{14}$$

In the following we will discuss the interaction problem for the anti-symmetric part of the solution. We start with the main deck $(\alpha, \beta) = (3,4)$, then using the upper deck (3,3)-layer we derive the interaction law and finally derive the lower deck (3,5)-layer problem. The anti-symmetric part of the pressure in the main deck can be expanded in the form

$$\Delta p = \Delta p_0^{(3,4)} + \mathrm{Re}^{-1/8} \Delta p_1^{(3,4)} + ..., \tag{15}$$

In contrast to (classical) triple deck problems the pressure is not constant across the main deck. The pressure involved in the interaction mechanism is of order $Re^{-1/8}$, i. e. $\Delta p_1^{(3,4)}$. The y-momentum equation reduces to

$$\frac{\partial \Delta p_0^{(3,4)}}{\partial y^{(4)}} = \kappa \bar{\theta}_0^{(3,4)}, \quad \frac{\partial \Delta p_1^{(3,4)}}{\partial y^{(4)}} = \bar{\theta}_1^{(3,4)}, \tag{16}$$

and using (14) we obtain

$$\Delta p_0^{(0,4)}(x^{(3)}, y^{(4)}) = \int_\infty^{y^{(4)}} \theta_B(\tilde{y}^{(4)})\, \mathrm{d}\tilde{y}^{(4)} + \Delta p_0^{(0,0)}(0,0), \tag{17}$$

$$\Delta p_1^{(0,4)}(x^{(3)}, y^{(4)}) = \bar{A}(x^{(3)}) \theta_B(y^{(4)}) + \Delta p_1^{(3,3)}(x_3, 0). \tag{18}$$

Since the flow in the upper deck is a potential flow with the velocity field

$$\Delta u - i\Delta v = \Delta u_0^{(0,0)}(0,0) + \mathrm{Re}^{-1/8} \left(\Delta u^{(3,3)}(x^{(3)}, y^{(3)}) - i\Delta v^{(3,3)}(x^{(3)}, y^{(3)}) \right) + ... \tag{19}$$

and $u_1^{(3,3)}(x^{(3)}, 0) = -\Delta p_1^{(3,3)}(x^{(3)}, 0)$, $v_1^{(3,3)}(x^{(3)}, 0) = -\Delta A'(x^{(3)})$ holds, the negative pressure $-p_1^{(3,3)}(x^{(3)}, 0)$ and the displacement thickness $\Delta A'(x^{(3)})$ can be interpreted as the real and imaginary part of a complex analytical function $\Delta\Phi_1$ evaluated on the real axis. We have

$$\Delta\Phi_1(x^{(3)}) = -\Delta p^{(3,3)}(x^{(3)}, 0) + i\Delta A'(x^{(3)}) = -\left(\Delta p^{(3,5)}(x^{(3)}) - \bar{A}(x^{(3)}) \right) + i\Delta A'(x^{(3)}). \tag{20}$$

Considering $\Delta p^{(3,5)}(x^{(3)}) = 0$ for $x > 0$ and using the asymptotic behavior of $\bar{A}$ for $x^{(3)} \to \infty$ we conclude that $\Delta\Phi_1(z) \sim (a + ib) z^{1/3}$ for $z \to \infty$ holds. The constants a and b are determined by using that $\Delta A' \to 0$ for $x^{(3)} \to -\infty$. They turn out to be $a = a_s$ and $b = -\sqrt{3} a_s$. Thus the asymptotic behavior of Δp_1 and $\Delta A'$ is given by

$$\Delta p_1^{(3,5)}(x^{(3)}) \sim -2a_s |x^{(3)}|^{1/3} \quad \text{for} \quad x^{(3)} \to -\infty, \tag{21}$$

$$\Delta A'(x^{(3)}) \sim -\sqrt{3} a_s |x^{(3)}|^{1/3}, \quad \text{for} \quad x^{(3)} \to \infty \tag{22}$$

and the interaction law can be written in the form:

$$\Delta A'(x^{(3)}) + \sqrt{3} a_s h(x^{(3)}) \left(x^{(3)} \right)^{1/3} =$$

$$-\left[\frac{1}{\pi} \int_{-\infty}^{0} \frac{\Delta p^{(3,5)}(\xi) + 2a_s |\xi|^{1/3}}{x^{(3)} - \xi} \mathrm{d}\xi - \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\bar{A}(\xi) - a_s h(\xi) |\xi|^{1/3}}{x^{(3)} - \xi} \mathrm{d}\xi \right].$$

We have written the interaction law in form such that the singular parts are separated and the integrand in the Hilbert integral decays sufficiently fast to zero for $x^{(3)} \to \pm\infty$. The equations for the velocity profile in the lower deck are given by the momentum equation in x-direction

$$\bar{u}_1^{(3,5)} \frac{\partial \Delta u_0^{(3,5)}}{\partial x^{(3)}} + \Delta u_0^{(3,5)} \frac{\partial \bar{u}_1^{(3,5)}}{\partial x^{(3)}} + \bar{v}_1^{(3,5)} \frac{\partial \Delta u_0^{(3,5)}}{\partial y^{(5)}} + \Delta v_0^{(3,5)} \frac{\partial \bar{u}_1^{(3,5)}}{\partial y^{(5)}} = -\frac{\partial \Delta p_1^{(3,5)}}{\partial x^{(3)}} + \frac{\partial^2 \Delta u_0^{(3,5)}}{\left(\partial y^{(5)} \right)^2} \tag{23}$$

and the continuity equation. The boundary conditions are

$$\begin{aligned} \Delta u_0^{(3,5)}(x^{(3)},0) &= \Delta v_0^{(3,5)}(x^{(3)},0) = 0, \qquad x^{(3)} < 0 \quad \text{plate}, \\ \Delta u_0^{(3,5)}(x^{(3)},0) &= \Delta p_0^{(3,5)}(x^{(3)},0) = 0, \qquad x^{(3)} > 0 \quad \text{wake}. \end{aligned} \tag{24}$$

It remains to specify the asymptotic behavior of the velocity profile for $x^{(3)} \to -\infty$ and $y^{(5)} \to \infty$.

Considering the asymptotic behavior of the pressure $\Delta p_1^{(3,5)}$ and of $\bar{u}_1(3,5) \sim y^{(5)}$ for $x^{(3)} \to -\infty$ we conclude that the asymptotic behavior of the flow field in the lower deck is self-similar. Using a (scaled) stream function ΔH defined by

$$\Delta u^{(3,5)} = \Delta H'(\eta), \quad \text{with} \quad \eta = \frac{y^{(5)}}{|x^{(3)}|^{1/3}} \tag{25}$$

we obtain the similarity equation for ΔH

$$3\Delta H''' - f_B''(0)\left(\eta^2 \Delta H'' - \eta \Delta H' + \Delta H\right) = 2a_s \tag{26}$$

with the boundary conditions $\Delta H(0) = \Delta H'(0) = 0$. The corresponding homogeneous equation has three linearly independent solutions $h_1(\eta) \sim \eta \ln \eta$ for $\eta \to \infty$, $h_2(\eta) = \eta$ and $h_3(\eta)$. The third solution h_3 increases exponentially. In order to match the velocity profile with the main deck solutions h_3 has to be eliminated. Thus we have

$$\Delta H(\eta) = -\frac{2a_s}{f_B''(0)} + c_1 h_1(\eta) + c_2 \eta \sim -2a_s + c_1 \eta \ln \eta + c_2 \eta, \quad \eta \to \infty. \tag{27}$$

Since there are two boundary conditions at $\eta = 0$ the constants c_1 and c_2 are uniquely defined. For the velocity profile we obtain

$$\Delta u_0(x^{(3)}, y^{(5)}) \sim \Delta H(\eta) \sim c_1 \ln y^{(5)} - \frac{c_1}{3} \ln |x^{(3)}| + c_1 + c_2, \quad x^{(3)} \to -\infty, \quad y^{(5)} \to \infty. \tag{28}$$

To supplement the lower deck equation with correct asymptotic boundary condition for $y^{(5)} \to \infty$ we need a condition which is satisfied by all linear combinations of the two admissible fundamentals solutions 1 and $\ln y^{(5)}$. Such a condition is given by

$$y^{(5)} \frac{\partial^2 \Delta u^{(3,5)}}{\partial \left(y^{(5)}\right)^2} + \frac{\partial \Delta u^{(3,5)}}{\partial y^{(5)}} \to 0, \quad \text{for} \quad y^{(5)} \to \infty. \tag{29}$$

The $y^{(5)}$ independent part of the asymptotic behavior of $u^{(3,5)}$ can be interpreted as the asymptotic behavior of the negative displacement thickness ΔA. Thus we have

$$\Delta A\left(x^{(3)}\right) \sim (c_1 + c_2) - \frac{c_1}{3} \ln |x^{(3)}|. \tag{30}$$

Matching the lower deck velocity profile with the main deck velocity profile we obtain

$$\Delta u^{(3,4)} \sim \ln \mathrm{Re} \, \frac{c_1}{f_B''(0)} f_B''(y^{(4)}) + \Delta A(x^{(3)}) f_B''(y^{(4)}) + \Delta u^{(0,4)}(0-, y^{(4)}) + \ldots \quad . \tag{31}$$

In the boundary layer the velocity profile close to the trailing edge $0 < -x^{(0)} \ll 1$ is given by

$$\Delta u^{(0,4)} = \begin{cases} H'\left(\frac{y^{(4)}}{\left(x^{(0)}\right)^{1/3}}\right) & \text{for} \quad (y^{(4)})^3 \sim |x^{(0)}| \\ \Delta u^{(0,4)}(0-, y^{(4)}) - \frac{c_1}{3f_B''(0)} \ln |x^{(0)}| f_B''(y^{(4)}) + \ldots & \text{for} \quad (y^{(4)})^3 \gg |x^{(0)}| \end{cases}, \tag{32}$$

where matching the ln-terms in of the main deck and lower deck yields the asymptotic behavior of $u^{(0,4)}(0-, y^{(4)}) \sim c_1 \ln y^{(4)}$ for $y^{(4)} \to 0$.

5. Numerical solution

The lower deck equations with the interaction law for the anti-symmetric part of the flow field are solved using Veldman's iteration method [3]. In $x^{(3)}$ a stretched grid has been used $x_{i+1} - x_i = f(x_i - x_{i-1})$ for $1 < i < N$, and $x_{-i} = -x_i$. The minimal step size is $x_i = 0.001$, the factor $f = 1.01$ and $N = 300$. Thus computational domain is $(-10^7, 10^7)$. At x_{-N} the similarity solution for the velocity profile is described. The momentum equation (23) is discretiazed with backward differences with respect to x. At each node x_i an ordinary differential equation is obtained which is solved using the ODE solver COLPAR [5].

In figure 2 the negative displacement thickness ΔA and the interaction pressure $p^{(3,5)}$ for the anti-symmetric part of the flow field are shown. The asymptotic behavior for $x \to -\infty$ of ΔA and $\Delta p_1^{(3,5)}$ is shown on figure 2b on a logaritmic and double logarithmic scale, respectively.

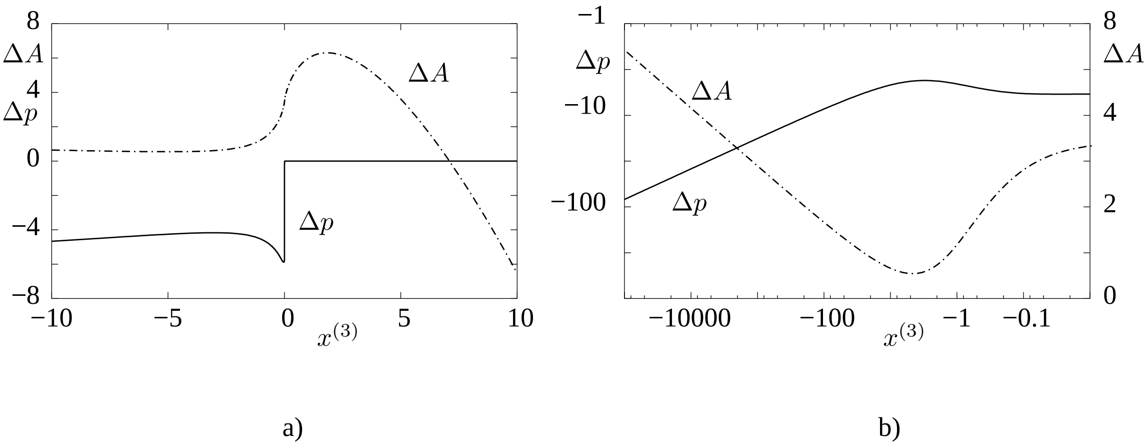

Figure 2: Negative displacement thickness ΔA and interaction pressure $\Delta p^{(3,5)}$ a). Asymptotic behavior of ΔA, and $\Delta p^{(3,5)}$ for $x^{(3)} \to -\infty$ b).

It turns out that the interaction pressure $p_1^{(3,5)}$ has a jump discontinuity at the trailing $x = 0$. In the next section sub-layers will be introduced to resolve this discontinuity.

6. Additional sub-layers

Due to the discontinuity of the difference pressure $\Delta p_1^{(3,5)}$ in the lower deck at the trailing edge the difference pressure has a discontinuity in the main deck as well. In the upper deck the difference pressure $\Delta p^{(3,3)}$ is singular at $(0, 0)$. Using the calculus of analytic functions of a complex variable $z^{(3)} = x^{(3)} + iy^{(3)}$ we can guess the behavior of $\Delta p^{(3,3)}$ close to 0:

$$\Delta p \sim \Delta p^{(0,0)} + \mathrm{Re}^{-1/8} \Delta p_1^{(3,3)}(x^{(3)}, y^{(3)}), \quad \Delta p_1^{(3,3)} \sim -\bar{A}(0) + \frac{[\Delta p^{(3,5)}]}{\pi} \Re\, i \ln z, \tag{33}$$

where $\Re z$ denotes the real part of a complex number and $[\Delta p^{(3,5)}] = -\Delta p^{(3,5)}(0-)$ is the jump of $\Delta p^{(3,5)}$ at the discontinuity at $x = 0$.

In order to resolve the discontinuity in the main deck we introduce the (4,4)-sub layer in the main deck. Using the expansion for the anti symmetric part

$$\Delta u \sim \ln \mathrm{Re} \frac{c_1}{f_B''(0)} f_B''(y^{(4)}) + \Delta u_0^{(3,4)}(0, y^{(4)}) + \mathrm{Re}^{-1/8} \ln \mathrm{Re} \frac{[\Delta p]}{8\pi} x^{(4)} f_B''(y^{(4)}) + \tag{34}$$

$$+ \mathrm{Re}^{-1/8} \Delta u_1^{(4,4)}(x^{(4)}, y^{(4)}) + ...,$$

$$\Delta v \sim -\mathrm{Re}^{-1/8} \ln \mathrm{Re} \frac{[\Delta p]}{8\pi} f_B'(y^{(4)}) + \mathrm{Re}^{-1/8} v_1^{(4,4)}(x^{(4)}, y^{(4)}) + ... \tag{35}$$

$$\Delta p \sim \Delta p_0^{(0,0)}(0,0) + \mathrm{Re}^{-1/8} \Delta p_1^{(4,4)}(x^{(4)}, y^{(4)}) + ... \tag{36}$$

we obtain the following equations for the leading order terms.

$$f_B' \frac{\partial \Delta u_1^{(4,4)}}{\partial x^{(4)}} + \Delta v_1^{(4,4)} f_B'' = -\frac{\partial \Delta p_1^{(4,4)}}{\partial x^{(4)}} \tag{37}$$

$$f_B' \frac{\partial \Delta v_1^{(4,4)}}{\partial x^{(4)}} = -\frac{\partial \Delta p_1^{(4,4)}}{\partial y^{(4)}} + \bar{\theta}_1^{(0,4)} \tag{38}$$

$$\frac{\partial \Delta u_1^{(4,4)}}{\partial x^{(4)}} + \frac{\partial \Delta v_1^{(4,4)}}{\partial y^{(4)}} = 0 \tag{39}$$

The flow in the (4,4)-sub-layer is inviscid but in contrast to the main deck the y-momentum equation is not degenerated. Eliminating $\Delta u_1^{(4,4)}$ and $\Delta v_1^{(4,4)}$ an elliptic equation for $\Delta p_1^{(4,4)}$ can be derived

$$f_B' \left[\frac{\partial^2 \Delta p_1^{(4,4)}}{\partial \left(x^{(4)}\right)^2} + \frac{\partial^2 \Delta p_1^{(4,4)}}{\partial \left(y^{(4)}\right)^2} - \frac{\partial \bar{\theta}_1^{(0,4)}}{\partial y^{(4)}} \right] + 2 f_B'' \left[\bar{\theta}_1^{(0,4)} - \frac{\partial \Delta p_1^{(4,4)}}{\partial y^{(4)}} \right] = 0. \tag{40}$$

The boundary and matching conditions can expressed as

$$\Delta p_1^{(4,4)} \sim -\frac{[\Delta p^{(3,5)}]}{\pi} \arctan \frac{y^{(4)}}{x^{(4)}} + \bar{A}(0) \left(\theta_0(y^{(4)}) - 1 \right), \tag{41}$$

for $y^{(4)} = 0$ or $r^{(4)} = \sqrt{(x^{(4)})^2 + (y^{(4)})^2} \to \infty$. For the numerical solution we decompose the solution of the linear elliptic partial differential equation (40) into a particular solution and a solution of the homogenous problem:

$$\Delta p_1^{(4,4)} \sim -\frac{[\Delta p^{(3,5)}]}{\pi} \Delta p_h^{(4,4)}(y^{(4)}, x^{(4)}) + \bar{A}(0) \left(\theta_0(y^{(4)}) - 1 \right), \tag{42}$$

with $\Delta p_h^{(4,4)} \sim \arctan y^{(4)}/x^{(4)}$ for $(x^{(4)})^2 + (y^{(4)})^2 \to \infty$ and $\Delta p_h^{(4,4)}(x^{(4)}, 0) = \pi$ for $x < 0$ and $\Delta p_h(x^{(4)}, 0) = 0$ for $x > 0$.

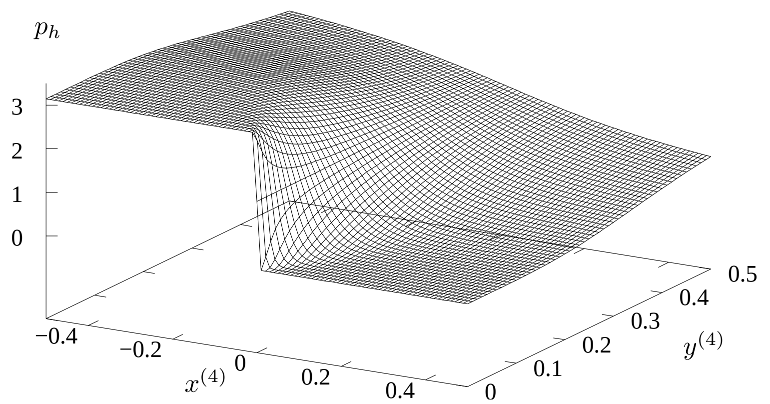

Figure 3: Local behavior of the interaction pressure $\Delta p_1^{(4,4)}$ near the trailing edge. The solution p_h of the homogenous problem , cf. (42), is shown

The local behavior near the singularity can be discussed by transforming the equation (40) to polar coordinates $r^{(4)}, \varphi$. Expanding $\Delta p_h^{(4,4)} \sim \Delta p_{h,0}(\varphi) + O(r^{(4)})$ for $r^{(4)} \ll 1$ we obtain

$$\sin\varphi\,\Delta p''_{h,0} - 2\cos\varphi\Delta p_{h,0} = 0, \quad \Delta p_{h,0}(0) = 0, \quad \Delta p_{h,0}(\pi) = \pi, \tag{43}$$

with the solution

$$\Delta p_{h,0} = \varphi - \frac{1}{2}\sin 2\varphi. \tag{44}$$

A numerical solution for p_h is shown in figure 3. The correct asymptotic behavior for $r^{(4)} \to 0$ and $r^{(4)} \to \infty$ could be verified.

We note that a similar analysis can be performed in the (5,5) sub-layer of the lower deck. In this layer the pressure is continuous with the exception of the point $(0,0)$. We believe that a complete resolution of the singular behavior of the pressure can only be obtained on the (6,6)-scale where the flow is described by the full Navier-Stokes equations.

7. Conclusions

A complete asymptotic analysis of the mixed convection flow around a finite horizontal plate under a small angle of attack in the limit of large Reynolds number and small buoyancy effects has been performed. Near the trailing edge a the flow is described by a triple deck problem. On triple deck scales the pressure turned out to be discontinuous at the trailing edge thus sub-layers in the main and lower deck have been introduced. Although on triple deck scales there is a pressure jump a trailing edge on the scales of the potential flow (leading order) the pressure is continuous at the trailing edge satisfying the Kutta condition.

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