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A Set-packing Approach to Routing Trains Through Railway Stations

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Abstract

Routing trains through a railway station consists in the assignment of a set of trains to routes that pass through a railway station or railway junction. This problem occurs on the strategic, tactical, and operational planning levels with different goals. A set-packing model is proposed and column generation and a constraint branching technique are used to solve it. Good results were obtained for a test example from the literature.

1 Introduction

In many European countries railways play an improtant role in the public and freight transport sytems. Extensive rail networks exist and major stations and railway junctions have a complex infrastructure. A problem that arises in this context is that of assigning a set of trains to routes through a railway station or junction over a period of time. The route assignment requires that no pair of trains is in conflict, i.e. that no pair of trains is scheuled to use the same track section at any point in time.

The layout of a railway station consists of a set of track sections which can be used by the trains. Trains enter the railway station through track sections which are part of a group of *entering points* and they leave through track sections which belong to a group of *leaving points*. Within these points, the railway infrastructure consists of *normal track sections* and *platform track sections*, a sequence of which defines a *route*. This route is associated to a timetable – which for the routing problem is assumed to be given – that consists of an *arrival time* and a *departure time* at a platform track section.

The train routing problem (TRP) occurs in three planning levels (see [9]). Depending on the planning goal, it can be of either *strategic, tactical*, or *operational* nature. The strategic planning level gives an answer to future capacity requirements of a station: given a current set of trains being routed through a station, the *feasibility problem* is solved for an expected traffic increase in future years. The feasibility problem consists in determining whether all trains scheduled in a timetable can be assigned a route through a station with its given layout. The tactical planning level deals with the present time and the actual generation or validatin of timetables for the trains that go through the railway station. Finally, at the operational level, day-to-day disturbances (such as delayed trains) and their effect on current timetables are considered and resolved such that all scheduled trains make their way through the railway junction.

Previous approaches to deal with the train routing problem include the research done by [3] who solve it as a graph coloring problem on special graph classes. Graph coloring and integer programming strategies can be found in the work of [2]. Closer to the contents of this paper is the modelling of the TRP as a set-packing problem (SPP) which [9] make use of at the strategic planning level, while [4] and [8] apply it to the tactical planning level.

The main objective of this paper is to model the problem in a way that is analogous to set partitioning models for crew rostering problems. This representation allows us to solve the problem in all three planning level hierarchies. In the next two sections, the formulation will be described and the solution approach will be presented.

2 The Train Routing Problem

The main difference between the model of [4] and [8] and the approach presented in this paper consists in the way conflict situations are modelled. In the TRP conflict is a scenario in which two or more trains compete for a a track section at the same moment in time. [4] and [8] define tuples of train-route pairs which are in conflict. By allowing only one of the two conflicting routes of trains to be assigned, a solution to the TRP will be conflict free. In the formulation presented in this section, the conflict is avoided by allowing each track section to be part of at most one route for one train at any one time. In fact, time is discretized to some reasonably chosen interval length (e.g. one second, 15 seconds, or one minute).

For this, let $\mathcal{T}$ represent the set of trains that need to be routed through the railway station. Furthermore, let $\mathcal{S}$ represent the set of track sections given by the layout of the railway station, and $\mathcal{H}$ be the set representing the time blocks corresponding to the discretization of the planning horizon. Moreover, let

$$1_{(t,r)^{(h,s)}} = \begin{cases} 1 & \text{if train } t \text{ on route } r \text{ crosses track section } s \text{ at time block } h \\ 0 & \text{otherwise} \end{cases}$$

and define the decision variables

$$x_{(t,r)} = \begin{cases} 1 & \text{if train } t \text{ follows route } r \text{ through the railway station} \\ 0 & \text{otherwise.} \end{cases}$$

With cost coefficients $c_{(t,r)}$ representing a preference for a route r of train t, the mathematical programming model for the TRP becomes

$$\max \sum_{t=1}^{|\mathcal{T}|} \sum_{r=1}^{n_t} c_{(t,r)} x_{(t,r)} \tag{1}$$

$$\text{subject to} \sum_{r=1}^{n_t} x_{(t,r)} \leq 1 \qquad \text{for all } t \in \mathcal{T} \tag{2}$$

$$\sum_{t=1}^{|\mathcal{T}|} \sum_{r=1}^{n_t} 1_{(t,r)^{(h,s)}} x_{(t,r)} \leq 1 \qquad \text{for all } s \in \mathcal{S}, h \in \mathcal{H} \tag{3}$$

$$x_{(t,r)} \in \{0,1\}, \tag{4}$$

where n_t is the number of possible routes for train t.

(1) maximizes preference of train routes (or number of trains if $c_{(t,r)} = 1$) scheduled through the railway station.

(2) are *train constraints* which ensure that only one route is assigned to every train.

(3) are *track section constraints* which guarantee that each track section at a period of time is only used by one train.

(4) defines the binary character of the decision variables.

The analogy to crew rostering models now becomes apparent: In crew rostering, $\mathcal{T}$ would be a set of crew members, $\mathcal{S} \times \mathcal{H}$ would be a set of tasks to be assigned to crew. (3) with equality constraints then guarantees that every task is assigned to a crew member, and (2) with equality constraints ensures each crew member needs to carry out exactly one set of tasks. The variables in the model correspond to sets of tasks that can be performed by a particular crew member, e.g. a line of work consisting of a number of tours of duty in airline crew scheduling [6].

Note that the TRP is an SPP and that the coefficient matrix has a distinguishable structure, both column and row-wise. Row-wise, the two main blocks are the previously mentioned train and track section constraints while column-wise, the coefficient matrix consists of $|\mathcal{T}|$ different column blocks, one for every scheduled train. Finally, notice that the feasibility problem is simply the TRP formulation with $c_{(t,r)} = 1$ and it needs to be checked whether the optimal solution is equal to $|\mathcal{T}|$ or not, thus giving an affirmative or negative answer to the feasibility problem, respectively.

3 Solving the TRP

The SPP is an NP-complete problem according to [5] and thus no polynomial time algorithm is known. The solution approach described in this paper uses linear programming based branch and bound with column generation and constraint branching.

This builds on experience with solving large set partitioning problems in crew rostering [6]. It is known that the block of columns refering to a single train in constraints (2) and (3) defines a perfect matrix, and therefore no fractional solutions can occur with only a single block of variables.

Column generation is used to avoid enumeration of all possible routes for a train. Thus, the LP relaxation of (1) – (4) can be solved with a small set of decision variables. The restricted model starts with any subset of the decision variables of the master LP that contains a feasible solution. A pricing step follows that calculates the reduced cost of candidate columns of decision variables which are not part of the restricted model yet. Letting $a_{(t,r)}$ represent the coefficients for the column of the coefficient matrix of the master problem corresponding to train t and route r, and π be a vector of dual variables obtained after solving the restricted problem to optimality, the reduced cost for that column is given by

$$rc(a_{(t,r)}) = c_{(t,r)} - \pi^T a_{(t,r)}.$$

The column that yields the largest positive reduced cost is then added to the restricted problem and the pricing step repeated. When no candidate column leads to a positive reduced cost, then the solution x^* solves both the restricted and the master LP optimally. For the TRP, the column generation subproblem can be formulated as a shortest path problem.

In the previous step, a solution is found for a problem where the integrality constraints have been dropped. The TRP is, however, a binary program, thus in order to achieve integrality, a branch and bound procedure based on constraint branch is applied. Traditional variable branching is ineffective for our set-packing problems – the resolution of fractional solutions at the optimal solution of a relaxed SPP leads to a very large and unbalanced branching tree (see [7]). The constraint branching developed by [7] consists in identifying two constraints, $\hat{t}$ and $\hat{g}$, such that the following relation holds:

$$0 < \sum_{(t,r)\in J(\hat{t},\hat{g})} x_{(t,r)} < 1. \tag{5}$$

Here, the set $J(\hat{t}, \hat{g})$ is defined as the set of columns of the coefficient matrix which have non-zero coefficients for constraints $\hat{t}$ and $\hat{g}$ or $J(\hat{t}, \hat{g}) = \{(t, r)|a_{(\hat{t}(t,r))} = 1$ and $a_{(\hat{g}(t,r))} = 1\}$. In the TRP, any fractional solution will have at least one such pair of constraints (see [1]), in fact due to the perfect blocks in the constraint matrix, fractional variables can only occur due to two trains competing for the same track segment at the same time so that one of the two constraints can be selected from the train constraints (2) and the other from the track section constraint (3).

Having selected constraints $\hat{t}$ and $\hat{g}$, branching is enforced through the 1-branch ($\sum_{(t,r)\in J(\hat{t},\hat{g})} x_{(t,r)} = 1$) and the 0-branch ($\sum_{(t,r)\in J(\hat{t},\hat{g})} x_{(t,r)} = 0$), i.e. train $\hat{t}$ uses the track segment at the time or not. With this type of branching, a simultaneous elimination of many variables is achieved on each side of the branch, leading to a smaller tree. Moreover, as shown in [6], if $\hat{t}$ and $\hat{g}$ are chosen such that $\sum_{(t,r)\in J(\hat{t},\hat{g})} x_{(t,r)}$ is maximized (i.e. chosen as close to 1 as possible), a depth-first 1-branch will lead to a fast and good initial integer solution.

4 Tactical and Operational TRP

When the feasibility problem (strategic TRP) returns a negative answer, a simple way of trying to achieve feasibility is by taking individual routes within the current timetable and force them to enter the railway station a certain number of time periods before or after their initial schedule. This leads to the use of a time shift parameter δ, which was introduced by [9], and which describes the number of time blocks the arrival/departure time of a train is rushed or delayed. The maximum number of time blocks a train t is allowed to be rushed or delayed will be given by δ_t.

The previously defined decision variables are thus extended to $x^{\delta}_{(t,r)}$ and take the value of one if route r is selected for train t, shifting the train's given timetable δ units forward. The resulting formulation for the tactical TRP (t-TRP) is given by:

$$\max \sum_{\delta=-\delta_t}^{\delta_t} \sum_{t=1}^{|\mathcal{T}|} \sum_{r=1}^{n_t} c^{\delta}_{(t,r)} x^{\delta}_{(t,r)} \tag{6}$$

$$\text{subject to} \sum_{\delta=-\delta_t}^{\delta_t} \sum_{r=1}^{n_t} x^{\delta}_{(t,r)} \leq 1 \qquad \text{for all } t \in \mathcal{T} \tag{7}$$

$$\sum_{t=1}^{|\mathcal{T}|} \sum_{\delta=-\delta_t}^{\delta_t} \sum_{r=1}^{n_t} 1_{(t,r)^{(h,s)}} x^{\delta}_{(t,r)} \leq 1 \qquad \text{for all } h \in \mathcal{H}, s \in \mathcal{S} \tag{8}$$

$$x^{\delta}_{(t,r)} \in \{0,1\}. \tag{9}$$

The objective function used for the t-TRP aims at giving a preference to a resulting timetable that is as close as possible to the given but infeasible one. It penalizes a schedule proportional to its deviation from the original one as follows:

$$c^{\delta}_{(t,r)} = 1 - (0.1 \cdot |\delta|).$$

In practice, a feasible timetable is not always possible to achieve due to circumstances that arise along the itinerary of a train (e.g. malfunctioning of a locomotive, weather hazards). The resulting delays lead to possible infeasibility of the initially assigned routes of all trains, as they were feasible and optimal only for the scheduled arrival and departure times. Modelling the operational TRP (o-TRP), i.e. the TRP with delayed trains, is in essence a particular case of t-TRP. The outcome desires to achieve feasibility for a new timetable, namely the one composed of all on-time trains with their assigned routes and schedule (e.g. after solving the t-TRP), and the new arrival and departure times for the set of delayed trains.

In this sense, o-TRP consists of two phases: the first is a decision problem in which it has to be determined whether the delayed train can follow its assigned route despite the current delay. Should this yield a negative answer, then a new feasible assignment has to be looked for. This may involve enlarging the delay of already delayed trains, delay on-time trains or relocate routes for some or all trains. This delay is modelled in the same way as for the t-TRP, with the exception that trains can only be delayed with respect to their scheduled arrival/departure time as otherwise (rushing the arrival/departure time) boarding passengers are affected.

5 TRP – A Test Example

As mentioned in the introduction, the TRP deals with three major questions: first, the feasibility of assigning a set of trains through the railway station under a given timetable. Second, should the feasibility problem yield a negative solution, then the trains' timetable will be shifted such that a route can be assigned for all trains. Finally, the third question finds an answer to what the routes of all trains will be like, when a set of trains is delayed.

For the test case of the Pierrefitte-Gonesse railway junction – a big intersection between four major destinations in France – the route a scheduled train follows along the track sections is assumed to be given. Figure 1 shows the layout, track connections and directions a train may take to cross the railway junction.

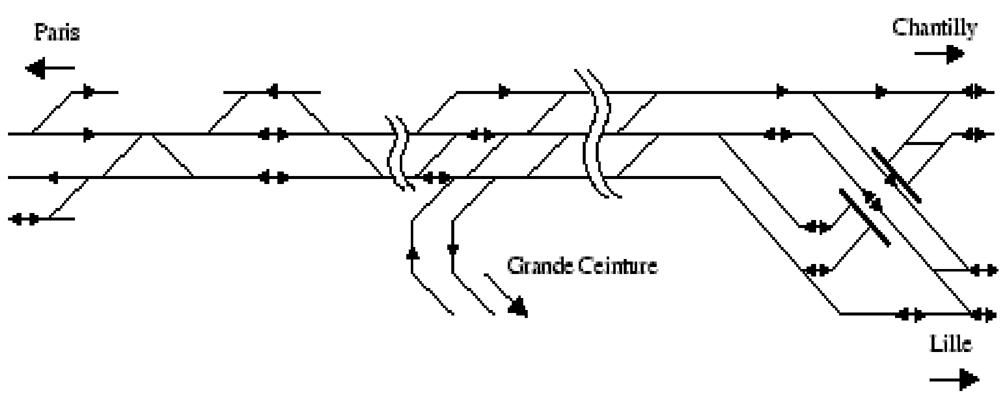

Figure 1: Layout of the test junction Pierrefitte-Gonesse (France).

As described before, the railway junction is subdivided into individual track sections. The 27 track sections for the test junction can be seen in Figure 2. Furthermore, the routes of the considered trains are those connecting Paris Gare du Nord and Lille (over track sections 1-18-2-3-5 and 16-22-17-25-9-14-19-15), Paris Gare du Nord and Chantilly (over track sections 1-18-2-3-4 and 11-20-12-23-10-13-14-19-15) and Grande Ceinture and Chantilly (over track sections 6-21-9-7-10-8-3-4 and 11-20-12-24-25-26-27).

The planning horizon for the test instance corresponds to 18.5 minutes. It is discretized in time blocks of 15 seconds duration, which means that the set $\mathcal{H} = \{0, 1, 2, \ldots, 74\}$ represents the time intervals $[0, 15), [15, 30), \ldots, [1110, 1125)$, with the time units being seconds. The travel time of a train on a particular

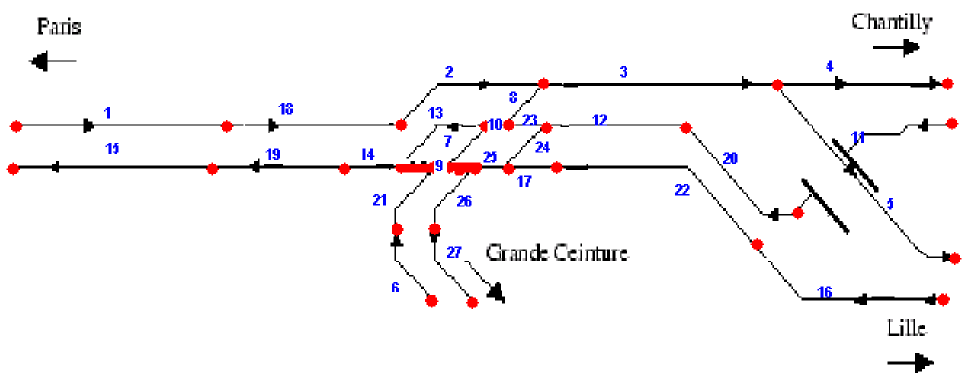

Figure 2: Layout of the junction Pierrefitte-Gonesse subdivided into track sections.

track section is fixed and given according to the speed limit imposed for each track section. It corresponds to the time span (set of time blocks) a track section will be set aside for a train to cross it, along its scheduled route. These values are given in Table 1.

Table 1: Travel times for trains crossing individual track sections of the Pierrefitte-Gonesse junction.

Track Section (No.)	Travel Time (Time Units)	Track Section (No.)	Travel Time (Time Units)	Track Section (No.)	Travel Time (Time Units)
1	6	10	2	19	6
2	6	11	6	20	6
3	4	12	1	21	4
4	4	13	2	22	4
5	6	14	1	23	1
6	4	15	6	24	4
7	4	16	6	25	1
8	4	17	1	26	4
9	1	18	6	27	4

As mentioned before, the TRP assumes that a timetable for all trains that need to be routed is given. Table 2 summarizes this input data for the test instance, identifying the eight trains that will cross the railway junction during the planning horizon as well as the origin and destination of the train. As Pierrefitte-Gonesse is a railway junction and not a station, arrival and departure times correspond to the time the train enters the first track section on its route, while the departure time corresponds to the time period in which it leaves this track section. Finally, the last column of Table 2 denotes the maximum number of time periods a train's scheduled arrival can be shifted either backward or forward. This data is used for the tactical planning level should the feasibility problem yield a negative answer. It provides degrees of freedom to alter, if necessary, a train's schedule.

Table 2: Train routing problem instance.

Train code	Origin (entering track No.)	Destination (leaving track No.)	Arrival Time	Departure Time	Slack time (Time Periods)
D1	Paris (1)	Chantilly (4)	12	18	2
D2	Lille (16)	Paris (15)	22	28	0
D3	Paris (1)	Lille (5)	23	29	6
D4	Chantilly (11)	Paris (15)	26	32	4
D5	Grande Ceinture (6)	Chantilly (4)	30	34	4
D6	Chantilly (11)	Paris (15)	33	39	2
D7	Lille (16)	Paris (15)	34	40	0
D8	Paris (1)	Lille (5)	36	42	2

6 Numerical Experience

All tests were run on a Pentium II PC with a 450 MHz Processor and 256 MB RAM. Given the timetable of Table 2, the feasibility problem was solved. Out of the eight trains, only train D7 could not be assigned to a route through the railway junction. The computational effort required was very small: the solution was obtained after only 0.3 seconds and the LP relaxation had an integer solution. This solution time is much faster than the computational time required by the formulation and solution approach in [4] for a nearly identical instance.

As it was not possible to assign a route to all scheduled trains under the current layout of the railway junction and the given timetable, the next question is whether the timetable can be altered such that the feasibility problem returns an affirmative answer.

To achieve this, the t-TRP was solved using the entries of column six of Table 2 as the maximum slack time or number of time periods which a train could have its schedule advanced or delayed. An optimal timetable that gives an affirmative answer for the feasibility problem was found and the results are given in Table 3. Here, column one states the code of the train, column two shows the arrival time under the original timetable, column three states the corrected arrival time which allows the feasibility problem to have an affirmative answer, and the fourth column lists the deviation of the new timetable from the original one. Recalling that train D7 was the train that led to the infeasibility of the initial timetable, it is interesting to note that in order to establish feasibility, two different trains had to be rescheduled. The computational effort was again very small, as it took 1 second to find the optimal solution. The optimal solution of the LP relaxation for this instance of the t-TRP was already integer, thus not requiring any branching.

After obtaining a feasible timetable from the previous step, two instances of o-TRP were considered to analyze the effect of a delay of trains on the resulting schedule. For the first, train D4 was set to be delayed by four time blocks, while for the second instance, trains D4 and D8 were set to be delayed by four time blocks. For each instance, it was assumed that all trains arriving after the first delayed train could be delayed if it were necessary.

Table 3: Modified timetable for the test instance.

Train	Original timetable (arrival time)	Modified timetable (arrival time)	Change
D1	12	12	0
D2	22	22	0
D3	23	23	0
D4	26	23	-3
D5	30	30	0
D6	33	35	2
D7	34	34	0
D8	36	36	0

The first phase returned a negative answer to the feasibility problem in both instances, meaning that under the current delay scenario, not all trains could be routed through the railway junction. This required solving the second phase of the o-TRP, which can lead to increasing the tobal delay in the system. The results of the second phase can be seen in Table 4, with columns three to five showing the results for the first of the instances and columns six to eight showing the results for the second instance. Columns four and seven show the corrected timetable which will allow all trains to be scheduled through the railway junction, taking into consideration the delays that had occurred before entering the railway junction (see columns three and six). Columns five and eight show the additional delay introduced to the system at the railway junction which is necessary to be able to route all trains through the railway junction. The CPU time for solving the second phase of the o-TRP was 0.3 seconds and again the LP relaxation optimal solution was integer.

Table 4: Results for the two test instances in which delayed trains were assumed.

		Instance A			Instance B		
Train	Scheduled timetable (arrival time)	Train Delay	Modified timetable (arrival time)	Added Delay	Train Delay	Modified timetable (arrival time)	Added Delay
D1	12	-	12	-	-	12	-
D2	22	-	22	-	-	22	-
D3	23	-	23	-	-	23	-
D4	23	4	27	0	4	27	0
D5	30	-	31	1	-	30	0
D6	35	-	39	4	-	39	4
D7	34	-	38	4	4	38	0
D8	36	-	37	1	-	36	0

7 Conclusions

A set-packing model was proposed for the TRP in its three planning levels. A solution method using column generation and constraint branching was implemented. A (small) example from the literature was used to test the proposed method. Determining the maximum number of trains that could be routed through the railway junction and proposing a modified timetable that would allow all trains to be routed was solved much faster than with the approach found in [4]. For the operational problem, two delay scenarios were modelled and solved very fast. Further research is directed towards applying the TRP formulation to larger railway stations or junctions as well as considering a larger planning horizon. Based on the experience gathered from this work and its promising results, solving larger TRP instances to optimality in relatively short time is expected.

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