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A customer-oriented approach

M. Michaelis and A. Schöbel

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Lotzestr. 16-18
D - 37083 Göttingen

Integrating Line Planning, Timetabling, and Vehicle Scheduling: A customer-oriented approach *

Mathias Michaelis   Anita Schöbel

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Abstract

Given an existing public transportation network, the classic planning process in public transportation is as follows: In a first step, the lines are designed; in a second step a timetable is calculated and finally the vehicle and crew schedules are planned. The drawback of this sequence is that the main factors for the costs (i.e. the number of vehicles and drivers needed) are only determined in a late stage of the planning process.

We hence suggest to reorder the classic sequence of the planning steps: In our new approach we first design the vehicle routes, then split them to lines and finally calculate the timetable. The advantage is that costs can be controlled during the whole process while the objective in all three steps is customer-oriented.

In the paper we formulate this approach, discuss the complexity of the resulting problems, and present a heuristic which we applied within a case-study, optimizing the local bus system in Göttingen, Germany.

1 Motivation and related literature

The strategic planning process in public transportation is usually divided in the planning steps depicted in Figure 1. In this paper we are interested in the following three steps: line planning, timetabling, and vehicle scheduling.

To sketch these three steps, let PTN $= (V, E)$ be a directed graph representing the public transportation network. It consists of a set of (potential) stops or stations V and a set of direct connections between them.

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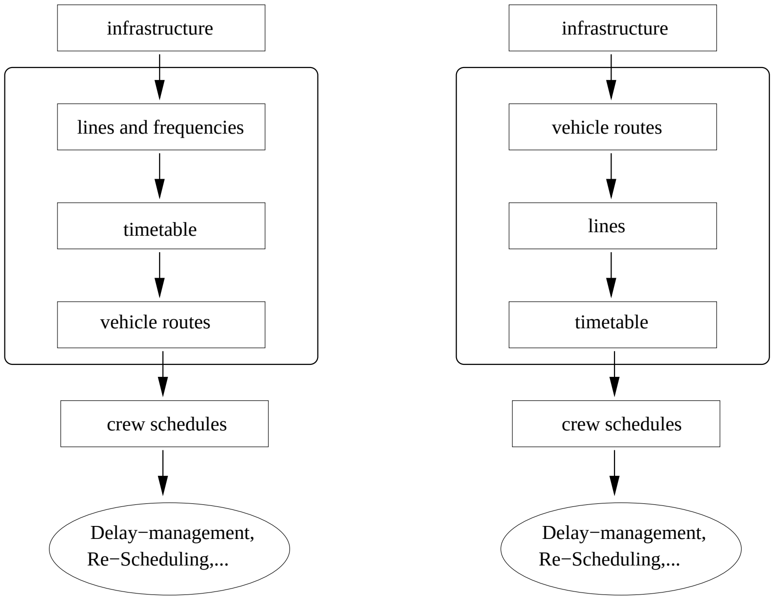

Figure 1: The classic planning phases in public transportation (left) compared to the sequence used in this paper (right).

Line planning. A *line* l is a path in the public transportation network PTN. The *frequency* f_l of a line l says how often service is offered along line l within a (given) time period I. A *line concept* is a set of lines together with their frequencies.
In most research papers it is assumed that a line pool of potential lines is already given. The goal is to choose a set of lines from the pool and to assign frequencies to the lines chosen. Unfortunately, even the feasibility problem (finding frequencies such that the constraints at each edge are satisfied) is NP hard (see [Bus98, CvDZ98]).
One distinguishes between cost-oriented models (see e.g. [CvDZ98, Zwa97, Goo04, BLL04, GvHK06]) in which the line concept has to cover a given demand with smallest possible costs, and customer-oriented models where a budget is given that should be used in a way that is "best" for the passengers. Examples for customer-oriented objective functions are to maximize the number of direct travelers ([BKZ96, Bus98]) or to minimize the traveling time of the passengers (see [BGP05, BP05, SS06, Sch05], where the latter two also took the time for transfers into account). Designing lines which can compete with the private mode has been studied in [LnMO06, LMO05]. Note that [CvDZ98] already considered the vehicle schedules of later planning steps.

There are rather few papers in which the lines are constructed during the process of line planning. In the very first paper about line planning, Patz ([Pat25]) starts with a line for each OD pair and iteratively eliminates lines by a greedy approach. A similar greedy heuristic is due to [Son77]. More recently, [UP95] and [Qua03] suggest constructive approaches, the latter also dealing with timetabling within the next planning step. Integration of line planning and periodic timetabling has also been done in [LM06].

In this paper we suggest a constructive heuristic using a customer-oriented approach.

Timetabling. Given the set of stations V and the set of vehicles F, a timetable consists of two functions $\pi^{arr} : V \times F \rightarrow \mathbb{N}$, $\pi^{dep} : V \times F \rightarrow \mathbb{N}$ assigning a departure time and an arrival time to each vehicle at each station. To avoid indices *event activity networks* are used in timetabling (see [Nac94]) in which the events consist of all arrivals and departures of all vehicles at all stations. The events are linked by edges corresponding to three types of activities: driving activities of vehicles between stations, waiting activities of vehicles at stations, and transfer activities to account for passengers changing busses or trains.

We have to distinguish between periodic and aperiodic timetabling. The latter can be efficiently solved by shortest path techniques while the former is NP-hard (see [Nac94]). The basis for tackling periodic timetabling is the *periodic event scheduling problem (PESP)* originally introduced in [SU89]. There are many extensive studies about timetabling, we refer to [Pee02, Lie06] and references therein. Current approaches deal with integration aspects (e.g. [LM04]) or robustness issues ([KDV07, LSS$^+$07, FSZ07]).

Vehicle Scheduling. If the lines and the timetable are given one can define the *trips* which have to be served, i.e. the minimal pieces which have to be operated by the same bus (usually between start and end station of a line). For each trip we have given its start station with its departure time and its end station with its arrival time. Two trips $trip_1$ and $trip_2$ can be served by the same bus if the arrival time at the end station of $trip_1$ plus the time needed to drive from the end station of $trip_1$ to the start station of $trip_2$ is smaller than the departure time at the start station of $trip_2$. The goal is to find a cost-minimal assignment between busses and trips such that each trip is covered by exactly one bus and the schedules of all vehicles are feasible. While the multi-depot case is NP-hard (see [BCG87] and [PDHH06] for a comparison of different heuristics), the single-depot case can be solved polynomially. Approaches include decomposition models ([Sah72]), assignment models ([Orl76]), transportation models ([GS78]) or network flow models ([DP95]). An excellent survey paper dealing with bus scheduling is [BK06], railway issues are treated in [Mar06].

Research in vehicle scheduling includes practical extensions as multiple vehicle types (e.g. [Löb97]), route constraints (e.g.[KGS06]), or maintenance issues. Recently, robustness issues are considered within the framework of ARRIVAL [ARR].

In contrast to the approaches in the literature and to the classic planning process in public transportation, we follow a new approach in this paper. We start by determining the routes of the vehicles, then add a timetable and split them to lines.

We repeat the most crucial notation that will be used throughout this text.

- A *line* is a path in the PTN along which service is offered.
- A *timetable* specifies the departure and arrival times of each vehicle at each station.
- For the vehicle schedules we distinguish between the *vehicle routes* which are given as paths in the PTN and the *vehicle schedule* which assigns arrival and departure times to the routes.

Since we are looking for a periodic schedule we assume that one common period T is given after which everything is repeated. We plan for only one period but take the periodicity into account when evaluating our objective function.

2 Planning an attractive transportation system

The main idea of our new approach is to start the whole process by designing the vehicle routes. A *vehicle route* is the path a vehicle drives in the PTN given as a sequence of stops in V or as a sequence of edges $e \in E$. The set of all routes in the final public transportation system is denoted by $\mathcal{U}$. Each vehicle route $u \in \mathcal{U}$ has a *frequency* f_u specifying how many trips should be offered along the route within the same planning period and a *schedule* t_u assigning an arrival and a departure time to each stop of the route. It will turn out that these values $(\mathcal{U}, f, t)$ are sufficient as variables, i.e, not only the vehicle schedules, but also the lines and the timetable together with their costs and attractiveness can be determined if $\mathcal{U}$ and f_u, t_u are known for all $u \in \mathcal{U}$.

We remark that the routes are planned as circles such that they can be repeated in the next period.
Let us consider the ingredients we need for the problem.

The public transportation network PTN =**(V,E).** For each edge e in the PTN we determine two lengths: $d_{bus}(e)$ is the time a bus needs for running between i and j, while $d_{priv}(e)$ is the time needed in the *private mode* i.e. by foot or by car. For most edges, $d_{priv}(e) \leq d_{bus}(e)$. The *duration* of a route is defined as the sum of all edge lengths (in the public mode) of edges contained in the route, i.e.

$$\mathrm{dur}(u) = \sum_{e \in u} d_{bus}(e).$$

Footpaths connecting nearby stops (e.g. on the two different sides of a street) are also included in our model to allow passengers to walk from one stop to another.

In our work we distinguish between *stops* V and *locations* $\mathcal{B}$, where the latter is a set of stops with the same name. Usually two stops (on either side of a road) form a location. In a one-way street there may be locations consisting of only one stop, whereas a location near an intersection may consist of four stops. The reason for aggregating the stops is that the evaluation of a public transportation system is based rather on locations than on stops since customers do not mind on which side of a street they depart or arrive.

Data about the potential demand. Our goal is to design an attractive public transportation system, i.e. one that meets the demand of the citizens. We are interested not only in improvements for existing customers but also in attracting new customers. Hence we use an origin-destination matrix representing the complete demand. This matrix is certainly not based on stops. It is given due to demand regions (called *cells*). By assigning cells to their closest locations we obtain an origin-destination matrix $\mathrm{OD} \in \mathbb{Z}^{|\mathcal{B}|\times|\mathcal{B}|}$. (Details are given in Section 4.) In the following let us assume that for each pair $i, j \in \mathcal{B}$ of locations the value OD_{ij} represents the number of persons who want to travel from i to j, i.e. the potential number of customers for this OD-pair.
Given an OD-pair of locations i, j a customer is interested in a "*good*" (i.e. a fast) trip from i to j in the public transportation system. These trips will be called *passengers' paths* between i and j.

Constraints. We consider two major constraints: the costs and the capacity of our system.
The costs of a public transportation system are mainly determined by the number of vehicles running per day, since this number determines not only the investment costs but also fixes the number of drivers and conductors needed. Our budget constraint hence bounds the number of vehicles $\mathbf{N}$ that we are allowed to use. Note that the number of vehicles needed (within one period of time) can be determined by the vehicle routes and their frequencies, namely by

$$\text{number of vehicles for route } u = \left\lceil \frac{\mathrm{dur}(u) \cdot f_u}{T} \right\rceil. \tag{1}$$

In our construction process we take care of designing vehicle routes u with a duration $\mathrm{dur}(u)$ a bit less than one time period T. In this case we obtain f_u as the number of busses necessary for route u.

There is another constraint we are taking into account: we ensure that the space available for busses is sufficient at each of the stops. As parameters we have given a capacity $cap(v)$ indicating how many busses are allowed to be at the stop v at the same time.

We remark that there are a lot of other constraints in practice. These include breaks for the drivers, slack times to make the timetable more robust and constraints for the specific shape and structure of the lines. They can be considered when constructing the vehicle routes in the first phase of our algorithm.

Objective function. We define the *attractiveness* of a public transportation system as the average probability that a (potential) traveler decides to use public transportation instead of the private mode. Our objective function hence is

$$\max \sum_{(i,j) \in \mathcal{B} \times \mathcal{B}} p_{ij} \mathrm{OD}_{ij} \tag{2}$$

where OD_{ij} is the potential demand between locations i and j and p_{ij} is the probability that a person who wants to travel between stops i and j uses public transportation. The probability p_{ij} depends on many factors. Talking to practitioners we decided to focus on

pw_{ij}: the average waiting time for trips from i to j and on

pd_{ij}: the travel time of public transport (compared to the travel time of the private mode) between i and j

to determine the probability that a person decides to use public transportation for his or her trip from i to j. The idea to compare the traveling times in public and private mode has also been used by Laporte, Mesa and Ortega, see [LMO05].

In the following we show in detail how to estimate p_{ij}. We start from a solution $(\mathcal{U}, f, t)$ consisting of vehicle routes $\mathcal{U}$ with their frequencies f and their schedules t.

We are interested in (the number and quality of) all possibilities how a passenger can travel from i to j. Given $(\mathcal{U}, f, t)$ such a *passenger path* is specified by

- the routes and stops it uses, and
- by the arrival and departure times of all its stops.

Note that two consecutive stops of a passenger path are either contained in the same route or the passenger has to transfer between two vehicles. In order to find all possible passengers' paths we set up the timetable graph defined by the PTN and our solution $(\mathcal{U}, f, t)$. This graph contains all the relevant information for a timetable information system and allows to determine the set of all possible passengers' paths $\mathcal{P}_{ij}$ from i to j for each pair of locations $i, j \in \mathcal{B}$, see [BDW07] for a recent comparison of methods.

For each $p \in \mathcal{P}_{ij}$ we collect

$$\begin{aligned} dep(p) &= \text{ starting time at } i \\ arr(p) &= \text{ arrival time at } j \\ \mathrm{dur}(p) &= arr(p) - start(p) \\ &= \text{time needed to travel from } i \text{ and } j \text{ using path } p \end{aligned}$$

We then take the best paths of this set. To this end we use the smallest possible traveling time

$$\mathrm{dur}_{ij}^{min} = \min_{p \in \mathcal{P}_{ij}} \mathrm{dur}(p)$$

and fix a value λ to determine

$$\begin{aligned} \mathcal{G}_{ij} \;=\; & \{p \in \mathcal{P}_{ij} : \mathrm{dur}(p) \leq \lambda \cdot \mathrm{dur}_{ij}^{min} \text{ and} \\ & \text{ there does not exist a path } p' \in \mathcal{P}_{ij} \text{ satisfying} \\ & dep(p') \geq dep(p), arr(p') \leq arr(p), \mathrm{dur}(p') \leq \mathrm{dur}(p)\} \end{aligned} \tag{3}$$

as the set of "*good*" passengers' paths between i and j. With the help of this set, we can estimate the two parameters *pd* and *pw* to estimate the probability that a customer uses public transportation when traveling from i to j:

pd: We compare the travel time in public transport with the travel time using the private mode, i.e. we calculate

$$r_{ij} = \frac{\mathrm{private}_{ij}}{\mathrm{public}_{ij}}$$

where $\mathrm{public}_{ij} = \frac{\sum_{p \in \mathcal{G}_{ij}} \mathrm{dur}(c)}{|\mathcal{G}_{ij}|}$ denotes the average travel time in public transportation and $\mathrm{private}_{ij}$ is the travel time in private transportation. The probability that a customer accepts public transportation is modeled by the following piecewise linear function (see left picture of Figure 2):

$$pd_{ij} = pd(r_{ij}) = \begin{cases} 1 & : \; r_{ij} \leq \alpha_1 \\ \frac{\alpha_2 - r_{ij}}{\alpha_2 - \alpha_1} & : \; \alpha_1 < r_{ij} \leq \alpha_2 \\ 0 & : \; r_{ij} > \alpha_2 \end{cases}$$

for two parameters α_1 and α_2.

pw: We determine the average waiting time wait_{ij} until the next trip in $\mathcal{G}_{ij}$ starts. To this end, we sort the passengers' paths in $\mathcal{G}_{ij}$ according to $dep(c)$ to obtain a list $dep(c_1) < dep(c_2) < \ldots < dep(c_K)$ with $k \leq |\mathcal{G}_{ij}|$. (Note that there are no paths with the same departure time in $\mathcal{G}_{ij}$.) This yields $K-1$ intervals

$$I_k = [dep(c_k), dep(c_{k+1})], j = k, \ldots, K-1.$$

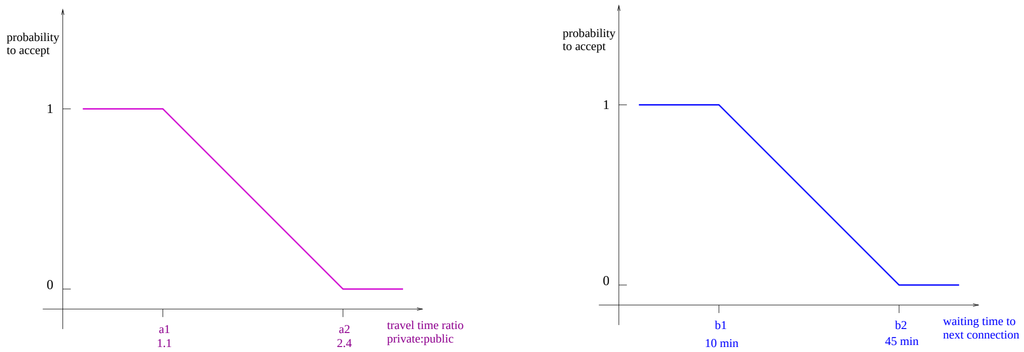

Figure 2: Probability for accepting the average waiting time and the ratio for the travel time for a path from i to j.

We assume that the demand is distributed evenly within a period, i.e. at each minute we have the same probability that a person wants to start his or her journey. If a person arrives within interval I_k, his or her average waiting time is $\frac{|I_k - 1|}{2}$ minutes. Hence we estimate

$$w_{ij} = \sum_{k=1}^{K} \frac{|I_k|(|I_k| - 1)}{2} \tag{4}$$

as the average waiting time for the next trip from i to j. Again, the probability that a customer accepts the average waiting time is modeled by a piecewise linear function (see right picture of Figure 2)

$$pw_{ij} = pw(w_{ij}) = \begin{cases} 1 & : \quad w_{ij} \leq \beta_1 \\ \frac{\beta_2 - w_{ij}}{\beta_2 - \beta_1} & : \quad \beta_1 < w_{ij} \leq \beta_2 \\ 0 & : \quad w_{ij} > \beta_2 \end{cases},$$

depending on the parameters β_1 and β_2.

Assuming that the probability pw_{ij} to accept the average waiting time is independent of the probability pd_{ij} to accept the travel time ratio, we finally get

$$p_{ij} = pw_{ij} \cdot pd_{ij}$$

and are hence able to calculate $att(\mathcal{U}, f, t)$ according to (2).

Note that the two functions depend on the customers' behavior which is represented by the parameters a_1, a_2, b_1, b_2 and λ.
In our case study these parameters are set to

- $a_1 = 1.1, a_2 = 2.5$ meaning that everybody accepts an increase of 10% of the travel time, but nobody would accept an increase by the factor 2.5,

- $b_1 = 7.5, b_2 = 36$, i.e. an average waiting time of 7.5 minutes (referring to a connection offered four times an hour) is accepted by all potential passengers, while an average waiting time of more than 36 minutes is not accepted at all. For public transportation at night we increased these values to 10 and 45.
- Due to Definition 3 of the set of good passengers' paths, λ has also an influence on the probability p_{ij}. In our case study we chose $\lambda = 1.3$.

Note that the specific values for the parameters have been chosen after discussion with practitioners. They make sense for the local properties of Göttingen, but need not hold in other environments. For example, in large cities, we suggest to choose smaller values for b_1 and b_2.

Our approach can now be summarized:

Phase 1: Design the routes $\mathcal{U}$ and the frequencies f of the vehicles.

Phase 2: Split the routes to lines.

Phase 3: Find a timetable t.

The three phases will be described in more detail in Section 5. We remark that splitting the vehicle routes to lines is just to obtain a nice graphical representation of the system, but has no influence on its attractiveness or on its costs (since the lines are not needed to calculate the costs or the shortest passengers' paths).

Summarizing, in our problem (P) we are looking for a set of vehicle routes $\mathcal{U}$, with frequencies $f_u \in \mathbb{N}$ for each $u \in \mathcal{U}$ and a timetable t_u for each $u \in \mathcal{U}$. A solution is denoted as $(\mathcal{U}, f, t)$. Our goal is to find a solution $(\mathcal{U}, f, t)$ with less than $\mathbf{N}$ vehicles minimizing $att(\mathcal{U}, f, t)$.

3 Complexity

Not very surprisingly, the integrated problem of planning lines, a timetable and the vehicle schedules is NP hard. More detailed, the following results hold.

Theorem 3.1.

- *It is NP-hard to design the routes of the vehicles, even if the timetable is not relevant, i.e. Phase 1 of (P) is NP-hard.*
- *It is NP-hard to find an optimal timetable, even if the vehicle routes are given, i.e. Phase 3 of (P) is NP-hard.*
- *The variant (P-special) in which all routes must contain a stop of a given central location, all frequencies have to be one, the set of edges with their lengths in the public and in the private mode coincide and the timetable is not relevant is still NP-hard.*

We present the proof for the third statement (which also proves the first.) The proof of the second statement can be found in [Mic07]; intuitively it also follows from the NP-hardness of periodic scheduling.

More formally, the third problem (P-special) can be described as follows:

(P-special) Given a PTN $= (V, E)$ with edge lengths $d(e) = d_{bus}(e) = d_{priv}(e)$ for each $e \in E$, a set of locations $\mathcal{B}$, a central location l_c and an origin-destination matrix OD, values $\lambda, a_1, a_2, b_1, b_2$ describing the users' behavior, a time period T, and two integers $\mathbf{N}$ and U, does there exist a solution $(\mathcal{U}, f, t)$ satisfying

- $l_c \cap u \neq \emptyset$ for all $u \in \mathcal{U}$,
- $f_u = 1$ for all $u \in \mathcal{U}$,
- $\sum_{u \in \mathcal{U}} \sum_{e \in \mathcal{U}} l(e) \leq \mathbf{N}$ (i.e. it can be run with $\mathbf{N}$ busses) and such that
- $\sum_{i=1} \sum_{j=1} p_{ij} OD_{ij} \geq U$?

Proof. We use a reduction from the knapsack problem which is known to be NP-hard (see [GJ79]). It is defined as follows: Given two natural numbers W, B and a set of items $\mathcal{D}$ with weights $w(d) \in \mathbb{N}$ and benefits $v(d) \in \mathbb{N}$ for all $d \in \mathcal{D}$, does there exist a subset $\mathcal{K} \subseteq \mathcal{D}$ of items with a total weight of no more than W and a total benefit of at least B?
Given an instance of (Knapsack), an instance of (P-special) is to be constructed. Define a central location l_c and a location l_d for each item in $d \in \mathcal{D}$ and add exactly one stop s_c and s_d for all $d \in \mathcal{D}$ for each of the locations. Connect all stops s_d star-wise to the central stop s_c with a pair of inverse edges. The lengths $l(e)$ of these two edges $e \in \{(s_d, s_c), (s_c, s_d)\}$ is set to $\frac{w(d) \cdot T}{2}$ for both the public and the private mode for each item $d \in \mathcal{D}$. We furthermore define the demand between the central location and the locations l_d as

$$\mathrm{OD}_{l_c, l_d} := v(d) \text{ for each } d \in D$$

and zero for all other pairs. For an illustration of this instance of (P-special) see Figure 3.

For the customers' behavior we set β_1 and β_2 so large that all waiting times will be accepted. Furthermore, we set $\alpha_1 \geq 1$ such that the customers accept the public mode if the traveling time is the same as in the private mode. This means that all existing paths are accepted by the passengers, independently of their timetables. Finally, we define $\mathbf{N} := W$ and $U := B$.

We now show that (P-special) has a feasible solution if and only if (Knapsack) has a feasible solution.

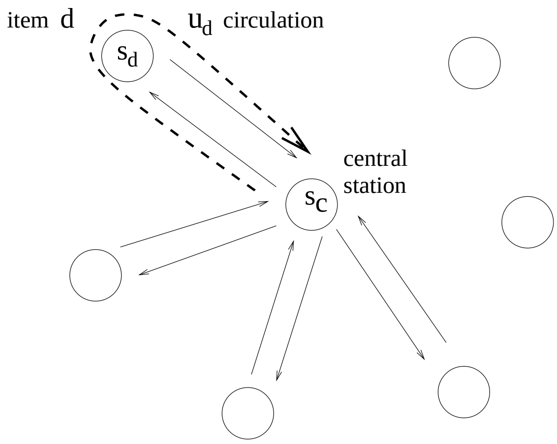

Figure 3: Reduction of (P-special) to (Knapsack).

(Knapsack) has a feasible solution: Let a feasible solution for (P-special) be given with a set $\mathcal{U}$ of routes. Every route contains the central stop l_c and at least one other stop. Without loss of generality we can assume that the route contains exactly one other stop (otherwise we split it to feasible routes for each other stop s_d it contains, since $2 \cdot \mathrm{dur}(e) \geq T$). We define $u_d := (s_c, s_d, s_c)$ as the route passing through stop s_d.

We now show that

$$K := \{d \in D : s_d \in u \text{ for some } u \in \mathcal{U}\} = \{d \in \mathcal{D} : u_d \in \mathcal{U}\}.$$

is a feasible solution of (Knapsack):

- The route u_d takes $2 \cdot \frac{w(d) \cdot T}{2} = w(d)T$ time. Hence, in order to run this route with a frequency of one, $w(d)$ busses are necessary, see (1). Since the solution $\mathcal{U}$ is feasible for (P-special) we conclude that
$$W \geq \left\lceil \frac{\sum_{u \in \mathcal{U}} \sum_{e \in u} l(e)}{T} \right\rceil = \sum_{d \in K} w(d).$$
- On the other hand, we know that the customers belonging to location l_d will use public transportation whenever $s_d \in u$ for some $u \in \mathcal{U}$, i.e. whenever $u_d \in \mathcal{U}$ exists. Together with the feasibility of the solution we obtain
$$B \leq \sum_{u \in \mathcal{U}} \text{demand covered by } u = \sum_{d \in K} v(d).$$

(P-special) has a feasible solution: Given a solution $\mathcal{K} \subseteq \mathcal{D}$ for (Knapsack), we construct a route $u_d := (s_c, s_d, s_c)$ with frequency $f_d = 1$ for each $d \in \mathcal{K}$ and set $\mathcal{U} := \{u_d : d \in K\}$. Then $\mathcal{U}$ satisfies the four conditions listed in the theorem:

- $s_c \in u$ for all $u \in \mathcal{U}$, hence $l_c \cap u \neq \emptyset$.
- $f_u = 1$ for all $u \in \mathcal{U}$.
- $\text{dur}(u) = \sum_{u \in \mathcal{U}} \sum_{e \in u} l(e) = \sum_{d \in K} \frac{2Tw(d)}{2}$ hence

$$\mathbf{N} \geq \text{number of vehicles } = \sum_{u \in \mathcal{U}} \left\lceil \frac{\text{dur}(u)}{T} \right\rceil = \sum_{d \in K} w(d)$$

  (i.e. it can be run with $\mathbf{N}$ busses)
- $U \leq \sum_{i=1} \sum_{j=1} p_{ij} OD_{ij} = \sum_{d \in K} v(d)$

Hence $\mathcal{U}$ is feasible for (P-special) and the proof is finished.

$\square$

4 Case Study

Before outlining our solution approach we describe the data of the case study we used. The case study was done within a cooperation with *Göttinger Verkehrsbetriebe* (GÖVB), the local bus company of Göttingen, Germany. The data we used consisted of 248 locations with 485 stops. The capacity of most of the stops is equal to four. It turned out that this is a crucial constraint: If left out we always obtained timetables in which up to 10 busses stopped simultaneously at the same station. We furthermore indicated the nodes that are in particular suitable for adding slack times for breaks.
As edges we used all edges contained in already existing lines, but we also added further edges representing streets which are currently not used by busses. The driving times of the new edges were fixed in cooperation with GÖVB. We also added footpaths between stops.
In order to estimate the traveling time in the private mode, we added additional edges which are not suitable for busses (e.g. if the streets are too narrow). The edge lengths in the private mode are usually shorter then in the public mode. An exception are some streets in the city center where we added additional time to account for the time-consuming task of finding a parking slot.

As demand data we received a partition of Göttingen into regions, called *cells* and data about the demand for each pair of cells. We assigned locations to cells (where a location can be assigned to more than one cell, and a cell can contain more than one location), estimated the importance of each location and expressed this by weights. Then we distributed the demand data to pairs of locations according to their assignment and weights.

An analysis of the current system showed its advantages and drawbacks: The driving times from the outskirts to the center are rather small. Moreover, twice an hour, many transfers are possible at one of the central stations. On the other hand, the capacity of this station is exceeded such that busses sometimes have to leave before the transferring passengers have arrived. We also noted that there are often long breaks at the end-stations of the lines (up to 20% of the duration of the route).

5 Solution Approach

According to Theorem 3.1, Phase 1 and Phase 3 of our solution approach are NP-hard by themselves. We therefore suggest to solve both of the problems heuristically. In the following we present the ideas we used. Some of them were motivated by the special requirements of Göttingen, but all of them can easily be adapted to other cities.

Given a solution $(\mathcal{U}, f, t)$ we can evaluate its objective value $att(\mathcal{U}, f, t)$ as shown in Section 2. As mentioned on page 9 we proceed in three steps. We first construct a reasonable set of routes, split them into lines and finally assign departure and arrival times to them.

Phase 1: Finding the vehicle routes with their frequencies

Each route is a circle in the public transportation network PTN. The basic idea of the algorithm is simple: We start with an arbitrary station s and move at random to one of its neighbors. We repeat this procedure until we end up at the starting station s again. Theoretically we can construct any route with this procedure, but in practice we have to guide it to obtain reasonable results. This can be done as follows.

Duration of a route. When generating the routes we keep the restrictions we have when adding departure and arrival times in mind. There are several reasons why some breaks (or additional slack time at stations) need to be added within the trips.
The most important one is to keep periodicity of the schedule. All vehicle routes should be repeated in each time period. Hence, the time needed for a route must satisfy

$$\mathrm{dur}(u) \cdot f_u = zT$$

for some integer z. To keep the unused time as small as possible we fix some (small) $\bar{\eta} > 0$ and only consider routes u with

$$\mathrm{dur}(u) \leq (z - \eta)\frac{T}{f_u} \tag{5}$$

where z is an integer and $0 < \eta < \bar{\eta}$. It is desirable that η is small, but not zero such that some additional slack time is available for each route. Such time can be used to provide slack times at stations in order to enable passengers to change to other busses, or more general, to make the

timetable robust against delays. It may also be needed for breaks for the drivers at the end stations. For each route the additional time η has to be distributed to the edge lengths. We propose to add such time to stopping times at stations where transfers are likely or to the stations farthest away from the center at turnaround activities.
In Göttingen, the period T equals 60 minutes. The restriction described here leads typically to routes with a duration of 60,90, or 120 minutes. The upper bound for η has been fixed to 10% of $\frac{T}{f_u}$.

Important stations. We identify a set of important locations and require that each route contains at least one of these stations. This significantly reduces the search space.
In Göttingen we declared two central locations as important. This means that all routes pass through the city center. This condition is justified since the demand between two non-central locations is rather small (according to the data we had and as expected due to gravity models).
Note that we have seen in part 3 of Theorem 3.1 that the problem remains NP-complete also with this reduced search space. Without loss of generality we can start the construction of a route u from such an important station. Let us call this station s_u in the following.

Other rules. One can set up many other restrictions or heuristics to construct and polish the routes found. Some of them are listed below. Let $\mathcal{U}$ be the set of routes already found.

- Stops that have not been covered by any other route of $\mathcal{U}$ should be more likely to be chosen such that we obtain a set of routes covering all stops. To this end one can weight the neighbors of the current stop s to increase the probability that a stop is chosen if it still does not appear in other routes. In our case study, we derived good results by weighting the unused stops by a factor of three.
- Circles within the routes should be avoided: This can be done by taking a new stop with a higher probability if it is not already in the route. (This rule is certainly not applied for the starting node s_u.)
- It may be desirable that routes contain most of their edges forward and backward (i.e. have a similar shape in inbound and outbound direction). To enforce this we suggest to consider only such routes in which the number of locations that consist of more than one station but only have one station in the route is small.
- In Göttingen we also implemented the following rule: Let us call a part of a route starting and ending at an important stop (in the city center) a *branch*. The public transportation company in Göttingen did not want to have routes with four or more branches. We took this into account by deleting all routes that visited the city center more than four times. This means that a station from the city center has

to appear between 25% and 75% of the (previously fixed) duration of the route. We used this observation to obtain a further reduction of the search space.

- Many other rules to model specific requirements are possible.

The algorithm is as follows. In each step we choose a time representing the duration of a route and a frequency as parameters. Then we construct a set of lines fitting to these two parameters. We evaluate the routes one by one and choose the best. The correct evaluation of the attractiveness requires a timetable which is not at hand during the first phase. Hence we estimate the objective function by setting all departure times at the (important) stop from which we started to zero. This ensures that passengers can transfer without large waiting times at these important stops. Summarizing, we obtain the following procedure.

Phase 1: Design of routes:

Step 1.1: $\mathcal{U} = \emptyset, n = \mathbf{N}$.

Step 1.2: Fix a frequency f_u and $\text{dur}^{fix} = \frac{z \cdot T}{f_u}$ for some integer z according to (5).

Step 1.3: Design a set of routes $u_1, \ldots u_h$ that include at least one important station with $\text{dur}^{fix} - \bar{\eta} \leq \text{dur}(u_k)x < \text{dur}^{fix}$ for $k = 1, \ldots, h$. One can require that the routes should respect some of the rules mentioned above.

Step 1.4: Add slack times to the edge lengths of u to obtain a duration of exactly dur^{fix}.

Step 1.5: Determine $u_i := \max_{j=1,\ldots,h} att(\mathcal{U} \cup \{u_j\}, f, 0\}$ and add $\mathcal{U} := \mathcal{U} \cup \{u_i\}$.

Step 1.6: $n := n - \frac{\text{dur}^{fix} \cdot f_u}{T}$

Step 1.7: If $n > 0$ goto Step 2.

Phase 2: Designing the lines

If the vehicle routes have been fixed we can represent them as lines. A line is a path through the PTN; hence each part of a route can be considered as a line. As lines are usually organized as tours it is preferable to take sub-circles of the routes.
As mentioned before, the representation by lines has no effect on where and when the busses drive and hence no effect on the objective function. Consequently, we can define the lines such that we get a “nice layout”.
In Göttingen, all routes have to pass through the city center. Moreover, no route is allowed to contain more than three branches. We hence chose

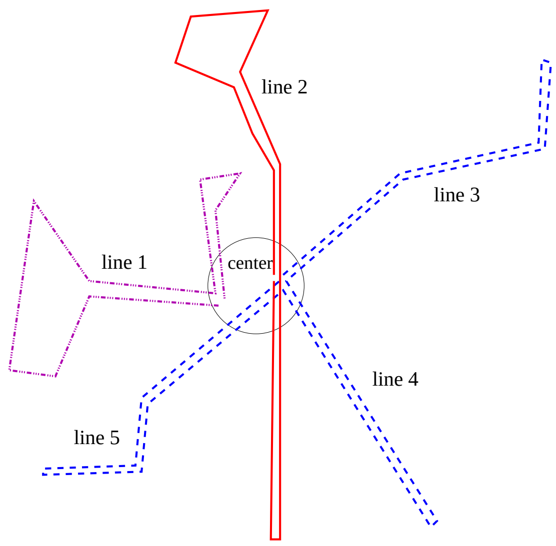

Figure 4: Three routes that are splitted to five lines.

branches or combinations of pairs of branches as lines, see Figure 4 for an illustration. These branches naturally are sub-circles of the routes. Algorithmically, we can proceed as follows.

Phase 2: Splitting routes to lines:

Input: $\mathcal{U}$

Step 2.1: For each route $u \in U$: Decompose U in circles. Choose the circles or unions of circles as lines.

Phase 3: Finding the timetable

As input for this phase we have given a set of routes $\mathcal{U}$ with their frequencies $f_u, u \in \mathcal{U}$. Our goal is to construct a feasible timetable. According to our constraints, a timetable is feasible if there is enough space at each of the stops in the system. We choose a timetable within the period $\{0, \ldots, T\}$ which is then repeated periodically. This is taken into account when evaluating our objective function att.

Since we already added slack time to the edges when constructing the routes, it is enough to fix one departure time for each route. We take the stop s_u from which we started to construct route u. A timetable is hence given as a vector $t \in \mathcal{T}^{|\mathcal{U}|}$ where $\mathcal{T} = \{0, 1, \ldots, T\}$ contains a discrete set of points in time (usually minutes). We call a timetable t optimal if

$$att(\mathcal{U}, f, t) = \max_{t' \in \mathcal{T}^{|\mathcal{U}|}} att(\mathcal{U}, f, t').$$

Consider a route u with frequency f_u and departure time t_u at stop s_u. Then another departures of the same route will take place at $t_u + z\frac{T}{f_u}$ for all integer values of z. Hence we only need to evaluate departure times $t_u \in \{0, 1, \ldots, \frac{T}{f_u}\}$. Even with this reduction it is not possible to try all possible combinations of departure times. Since Theorem 3.1 states that the problem of finding an optimal timetable is NP-hard we propose to use a heuristic also in this phase. The first idea to fix the departure times of each routes iteratively had the following drawback: We obtained routes, all departing at the same time from the same central station. When the capacity of this station was used, the next routes were placed very disadvantageous such that the final outcome was not really good.

We hence developed the following approach. We divide the routes into pairs and synchronize each pair in a first step. In a second step we combine the pairs to quadruples and synchronize them. We proceed in this manner until all routes are fixed. During this process we choose the pairs in each step by matching techniques to ensure that the most promising combinations are grouped.

More precisely, we define the following graph $G_{match} = (\mathcal{U}, E_{match})$ in which the nodes are defined as the routes $\mathcal{U}$ and we add an edge between two routes u_1, u_2 if $u_1 \cap u_2 \neq \emptyset$, i.e. if they contain at least one stop where a transfer is possible. As weight for edge $\{u_1, u_2\}$ we set

$$c_{u_1,u_2} := \max_{t_1,t_2 \in \mathcal{T}} att(\{u_1, u_2\}, \{f_{u_1}, f_{u_2}\}, (t_1, t_2)),$$

i.e. we choose the best possible synchronization of the two routes (independent of all other lines). Since one of the two times t_1, t_2 can arbitrarily be fixed we only have to evaluate

$$c_{u_1,u_2} := \max_{t \in \mathcal{T}} att(\{u_1, u_2\}, \{f_{u_1}, f_{u_2}\}, (0, t)) \qquad (6)$$

We then choose a cost-maximal matching in the graph G_{match} which synchronizes pairs of routes. Each of the pairs (or of single routes if the matching was not a perfect matching) is then clustered to one new node for the matching graph of the next step. In the second step we find an optimal matching of the groups and go on until only one group is left.

To state the algorithm we need to deal with groups of routes $g \subset \mathcal{U}$. Synchronizing such a group of routes means to find a timetable

$$t_g := (t_u : u \in g)$$

for all routes $u \in g$. Note that such a timetable can be shifted in time without changing its objective value, i.e.

$$att(g, f_u : u \in g, t_g) = att(g, f_u : u \in g, t_g + t)$$

where $t_g + t = (t_u + t : u \in g)$. We can hence assume without loss of generality that there is one representative route u_g in each group g with $t_{u_g} = 0$.
Given two two groups of routes g_1 and g_2 with two timetables t_{g_1} and t_{g_2}. If we want to synchronize these groups (without changing their internal timetables) we have to find

$$\max_{t \in \mathcal{T}} att(g_1 \cup g_2, (f_u, u \in g_1 \cup g_2), (t_{g_1}, t + t_{g_2}))$$

The optimal value for t is denoted as $t^*_{g_1,g_2}$ and called the *synchronization shift*.

Our algorithm starts with a first partition into groups, each group consisting of only one route. In each step, the groups are matched pairwise. (Some groups may be left unmatched if the matching is not perfect, but since the matching graph is nearly complete this is usually at most one group.)
The procedure can be summarized as follows.

Phase 3: Finding the timetable

Input: $\mathcal{U}$, f_u for all $u \in \mathcal{U}$.

Step 3.1: Define the first matching graph $G_{match} = (V_{match}, E_{match})$ with

- $V_{match} = \{\{u\} : u \in \mathcal{U}\}$
- $u_g = u$ if $g = \{u\}$ as representative route of group g
- $t_g = (0)$ as timetable of group g
- $E_{match} := \{\{g_1, g_2\}$: there exists $u_1 \in g_1, u_2 \in g_2$ such that $u_1 \cap u_2 \neq \emptyset\}$
- $c_{g_1,g_2} := \max_{t \in \mathcal{T}} att(g_1 \cup g_2, (f_u, u \in g_1 \cup g_2), (t_{g_1}, t + t_{g_2}))$ and let $t^*_{g_1,g_2}$ be the corresponding synchronization shift.

Step 3.2: Find a matching $E^m \subseteq E_{match}$ maximizing the sum of weights.

Step 3.3: Update groups: For each $e = \{g_1, g_2\} \in E^m$ define $g := g_1 \cup g_2$ and

- $V_{match} = V_{match} \cup \{g\} \setminus \{g_1, g_2\}$
- $u_g = u_{g_1}$ as representative route of group g
- $t_g = (t_{g_1}, t^*_{g_1,g_2} + t_{g_2})$ as timetable of group g using the synchronization shift calculated before.

Step 3.4: Update matching graph:

- $E_{match} := \{\{g_1, g_2\}$: there exists $u_1 \in g_1, u_2 \in g_2$ such that $u_1 \cap u_2 \neq \emptyset\}$
- $c_{g_1,g_2} := \max_{t \in \mathcal{T}} att(g_1 \cup g_2, (f_u, u \in g_1 \cup g_2), (t_{g_1}, t + t_{g_2}))$ and let $t^*_{g_1,g_2}$ be the corresponding synchronization shift.

Step 3.5: If $E_{match} = \emptyset$ stop. Output: $(t_g : g \in V_{match})$.
Otherwise goto Step 3.2.

After fixing a timetable with the above algorithm we used an improvement heuristic checking the distribution of the slack times which appear in equation (5) and have already been fixed in Phase 1. A redistribution may lead to further possibilities to transfer and hence further improve the objective function.

6 Results and Conclusion

We implemented our procedures and tested them within a case study in Göttingen. Our program needed 20 hours to generate a solution with 8 routes which we splitted to 10 lines. The solution improves the attractiveness of the current solution by 18.7%. The new timetable does not have

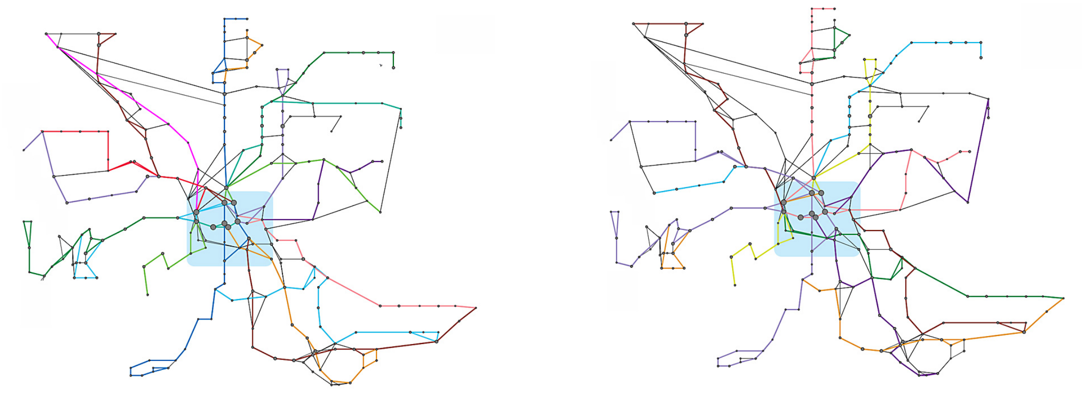

Figure 5: Comparison of new and old lines in Göttingen.

	System at night:	**System at daytime:**
current system.	23 busses, 11 routes	46 busses, 13 routes
"best" system	23 busses, 8 routes	42 busses, 12 routes
improvement	costs by 0%, att by 18%	costs by 10%, att by 1%

Table 1: The best solutions of our algorithm.

the long breaks at the ends of the lines and uses the additional busses to increase the frequencies of the routes. Moreover it is more robust due to the distribution of the slack times and it takes the capacities of the stations into account. The current lines and the new lines are shown in Figure 5. The figure shows that (nearly) all edges that have been covered by a route before are still covered. But the shape of the single vehicle routes changed, and also their durations and frequencies.

By decreasing the number of available busses we can also use the program to optimize the costs instead of the attractiveness of the system. This yielded a reduction of 10% of the busses and still increased the attractiveness by 1%. The two solutions which are best according to the practitioners of GÖVB are listed in Table 6.

At the moment, GÖVB is implementing the results in its new line system.

Summarizing, we presented a new integrated approach to tackle three problems in public transportation: line planning, timetabling and vehicle scheduling. We did not use the classical sequence of the planning phases but started by constructing the vehicle routes. Both phases, constructing the routes and fixing the timetable are NP-hard. In this paper we suggested heuristic solutions which worked very well in practice. However, we are sure that improvements can be made in both procedures and more theoretical results about these new types of problems can be obtained.

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Institut für Numerische und Angewandte Mathematik
Universität Göttingen
Lotzestr. 16-18
D - 37083 Göttingen

Telefon: 0551/394512
Telefax: 0551/393944

Email: trapp@math.uni-goettingen.de URL: http://www.num.math.uni-goettingen.de

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