JOURNAL OF INTEGRAL EQUATIONS
AND APPLICATIONS
Volume 22, Number 4, Winter 2010

CORRECTIONS TO NONLINEAR INTEGRAL EQUATIONS FOR SOLVING INVERSE BOUNDARY VALUE PROBLEMS FOR INCLUSIONS AND CRACKS JIEA 18 (2006), 13–38

O. IVANYSHYN AND R. KRESS

In the above paper the proof of Theorem 4.1 is false since the potential V_1 fails to be bounded. Here, we present a corrected proof. In what follows, we shall use the notations and the formula numbers of the original paper.

After noting that by the correct first part of the original proof we may assume that the perturbation vector has the form $\zeta_0 = q\,\nu$, we begin by deriving transformations of the derivatives $A_0'(r,\varphi;q)$ and $A_1'(r,\varphi;q)$. For this, for an arbitrary vector $a = (a_1, a_2)^\top$ we denote by $a^\perp := (-a_2, a_1)^\top$ the vector obtained by rotating a by 90 degrees in counter clockwise orientation. We start from the elementary relation

$$\operatorname{grad}_x \frac{\partial}{\partial\nu(y)}\,\Phi(x,y)\cdot a = \frac{\partial}{\partial\sigma(y)}\,\operatorname{grad}_x \Phi(x,y)\cdot a^\perp$$

where $y \in \Gamma_0$ and $x \in \mathbb{R}^2 \setminus \{y\}$ and σ is the unit tangent vector in counter clockwise orientation and ν the normal to Γ_0 directed into the exterior of D. Parameterizing $y = z_0(\tau)$ in view of $|z_0'(\tau)|\,\nu(z_0(\tau)) = [z_0'(\tau)]^\perp$ and $ds(z_0(\tau)) = |z_0'(\tau)|\,d\tau$ we find

$$(1)\quad \operatorname{grad}_x \left\{[z_0'(\tau)]^\perp \cdot \operatorname{grad}_y \Phi(x, z_0(\tau))\right\}\cdot a = \frac{d}{d\tau}\,\operatorname{grad}_x \Phi(x, z_0(\tau))\cdot a^\perp.$$

Choosing $x = z_0(t)$ and $a = \zeta_0(t) - \zeta_0(\tau)$, in view of the representation of $A_0'(r,\varphi;q)$ on the bottom of page 22 in the original paper yields

$$A_0'(r,\varphi;q)(t) = \int_0^{2\pi} \varphi(\tau)\frac{d}{d\tau}\left[\operatorname{grad}_x \Phi(z_0(t), z_0(\tau))\right]\cdot\left\{[\zeta_0(t)]^\perp - [\zeta_0(\tau)]^\perp\right\} d\tau$$
$$- \int_0^{2\pi} \varphi(\tau)\,\operatorname{grad}_x \Phi(z_0(t), z_0(\tau))\cdot[\zeta_0'(\tau)]^\perp\,d\tau,\ \ t\in[0,2\pi],$$

DOI:10.1216/JIE-2010-22-4-647

and from this it follows by partial integration that

$$A_0'(r,\varphi;q)(t) = \int_0^{2\pi} \varphi'(\tau)\,\mathrm{grad}_x\,\Phi(z_0(t),z_0(\tau)) \cdot \{[\zeta_0(\tau)]^\perp - [\zeta_0(t)]^\perp\}\,d\tau, \quad t\in[0,2\pi].$$

Here, we have used the fact that $\varphi = u\circ z_0$ is C^1 due to the regularity results under the homogeneous Neumann condition on Γ_0. We recall the definition (2.3) of the double-layer potential v with density φ and note from Maue's formula, i.e., from choosing $x = z_0(t)$ and $a = \zeta_0(t)$ in (1) and partial integration, that

$$\mathrm{grad}\,v(z_0(t))\cdot\zeta_0(t) = \int_0^{2\pi} \varphi'(\tau)\,\mathrm{grad}_x\,\Phi(z_0(t),z_0(\tau))\cdot[\zeta_0(t)]^\perp\,d\tau, \quad t\in[0,2\pi].$$

Hence, defining

$$W(x) := \mathrm{div}\int_0^{2\pi} \varphi'(\tau)\;\Phi(x,z_0(\tau))[\zeta_0(\tau)]^\perp\,d\tau, \quad x\in\mathbb{R}^2\setminus\Gamma_0,$$

we finally have

$$A_0'(r,\varphi;q) = W\circ z_0 - [\mathrm{grad}\,v\circ z_0]\cdot\zeta_0. \tag{2}$$

We note that both terms on the right hand side of (2) do not jump on Γ_0. Analogously, choosing $x = z_1(t)$ and $a = \zeta_0(\tau)$ in (1), we find

$$A_1'(r,\varphi;q) = W\circ z_1. \tag{3}$$

After further introducing the function

$$V(x) := \mathrm{div}\int_0^{2\pi} \psi(\tau)\Phi(x,z_0(\tau))\nu(z_0(\tau))\,d\tau, \quad x\in\mathbb{R}^2\setminus\Gamma_0,$$

in view of (3), the integral equation (4.6) implies that $V+W=0$ on Γ_1. Since V and W are bounded in D_1, from the uniqueness for the exterior Dirichlet problem and analyticity we can conclude that $V+W=0$ in $D\cup\overline{D}_1$. We recall the definition (2.2) of the combined potential w and note that, by Green's integral theorem, $v + w = 0$ in $\mathbb{R}^2 \setminus \overline{D}$. Then, in view of (2), the integral equation (4.5) implies that $V_+ + W_+ = 0$ on Γ_1 and consequently $V + W = 0$ in D_0, that is, finally $V + W = 0$ in $\mathbb{R}^2 \setminus \Gamma_0$. (Here, we have used the fact that the smoothing properties of the operators in (4.5) imply that ψ is continuous.)

Since W does jump across Γ_0 the jump relation for the double-layer potential V implies $\psi = 0$. After rewriting

$$W(x) = \int_0^{2\pi} \varphi'(\tau)\, q(\tau) \operatorname{grad}_x \Phi(z_0(t), z_0(\tau)) \cdot [\nu(z_0(\tau))]^\perp \, d\tau,$$
$$x \in \mathbb{R}^2 \setminus \Gamma_0,$$

with the aid of another partial integration we observe from the jump relations for the normal derivative of single-layer potentials that $\varphi' q =$ const. Since the derivative φ' of the periodic function φ has at least one zero, the constant must be zero. Now, assume that φ is constant on an open subset of Γ_0. Then by Holmgren's theorem we would have that u is constant in all of D. This contradicts the tacit assumption that f is not constant on Γ_1 (otherwise u is constant in D and the inverse problem makes no sense.) Hence, we also have $q = 0$, and the proof is complete. □