Local projection method for convection-diffusion equations

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We discuss the application of the finite element method to the numerical solution of scalar convection-diffusion or convection-diffusion-reaction equations. We are interested in the singularly perturbed case when the diffusivity is significantly smaller than the convection. Then the solution typically possesses interior and boundary layers whose widths are usually significantly smaller than the mesh size and hence the layers cannot be resolved properly. In particular, it is well known that the classical Galerkin finite element discretization is inappropriate in the convection-dominated regime since the discrete solution is typically globally polluted by spurious oscillations. Although, during the last three decades, an extensive research has been devoted to the development of methods which diminish spurious oscillations in the discrete solutions of convection-diffusion equations, their numerical solution is still a challenge when convection strongly dominates diffusion.

The aim of this talk is to discuss stabilization of convection-diffusion equations based on local projections. This technique has been introduced in [1] for the Stokes equations and become very popular during the last decade. An extension to transport problems was proposed in [2] and investigated in many papers later. Originally, the local projection method was designed as a two-level approach since the projection space was considered on a coarser mesh. Recently, a one-level approach was introduced in [3] based on using higher order polynomials rather than refining the triangulation. In the talk, we shall review theoretical results on both the one-level and the two-level approaches of the method and we shall present various numerical results illustrating the properties of the local projection method in the convection dominated regime.

References:

[1] R. Becker, M. Braack: A finite element pressure gradient stabilization for the Stokes equations based on local projections, Calcolo 38 (2001), 173-199

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[3] G. Matthies, P. Skrzypacz, L. Tobiska: A unified convergence analysis for local projection stabilisations applied to the Oseen problem, M2AN 41 (2007), 713-742