# LPS for Oseen problems: A general framework and the enrichment approach

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The local projection stabilisation is based on a projection from the underlying approximation space onto a discontinuous projection space which is defined either on a coarser mesh (resulting in the two-level method) or on the same mesh (giving the one-level or enrichment approach). We will concentrate on the latter one.

Stabilisation is derived from additional weighted $L2$-terms of the fluctuation of the gradient of pressure and velocity (or only parts of the latter like the divergence or the streamline derivative). The key point in the analysis is a suitable interpolation operator which fulfils not only optimal error estimates but has also additional orthogonality properties with respect to the projection space. We will discuss conditions which guarantee the existance of such interpolation operators.

The convergence properties of the enrichment approach applied to inf-sup stable discretisations are analysed in the second part of the talk. An interesting point is that for inf-sup stable finite element pairs we do not need an $H1$-stable interpolation operator with additional orthogonality properties for proving the stability of the discrete problem. As a consequence, there is much more flexibility for choosing the approximation and projection spaces. Most of the known inf-sup stable finite element pairs approximate the velocity components by elements of order $r$ and the pressure by elements of order $r-1$ which results in error estimates of order $r$. We propose new inf-sup stable finite element pairs approximating both velocity and pressure by elements of order $r$. In contrast to the 'classical' equal order interpolation, the velocity components and the pressure are discretised by different elements. We prove an error estimate of order $r+1/2$ uniformly in the viscosity.