On the stability of CIP and local projection methods

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ABSTRACT

We investigate the stability properties of numerical methods using higher order finite elements applied to a convection diffusion equation with a small diffusion parameter ε. As discretization mehtods we consider the standard Galerkin method and the stabilization methods based on CIP- and local projection techniques.

If we look at the stability of the discrete solution in the usual energy-type norm (including also extra terms coming from the stabilization) then we see that spurious oszillations over the whole domain are not prohibited by this norm. That means that the CIP-norm as well as the local-projection-norm do not "see" certain types of global oszillations.

Therefore, a new concept of *improved stability of a method* is presented which ensures that the discrete solution can have large gradients only on a subdomain with a measure of $O(h)$. For a one-dimensional model problem, it is shown that the CIP stabilization for third order finite elements with Dirichlet boundary conditions satisfies this improved stability only under the assumption that the CIP-parameter γ is sufficiently small. In the general d-dimensional case, $1 \leq d \leq 3$, it is shown that the CIP stabilization for higher order finite elements with Nitsche type boundary conditions satisfies the improved stability for all CIP-parameters (even for the Galerkin case with $\gamma = 0$). A numerical study on the size of the stability constants is presented for the CIP- and local projection stabilization.