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The Multivariate Bernstein Basis Polynomials and Their Kernels

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Abstract

We consider the *d-variate Bernstein basis polynomials of degree n*

$$B_\alpha(x_1,\ldots,x_d) := \binom{n}{\alpha}\,\mathbf{x}^\alpha := \frac{n!}{\alpha_0!\alpha_1!\ldots\alpha_d!}\,x_0^{\alpha_0}x_1^{\alpha_1}\cdots x_d^{\alpha_d}\ ,$$

for $\alpha = (\alpha_0,\alpha_1,\ldots,\alpha_d) \in \mathbb{N}_0^{d+1}$ with $|\alpha| := \alpha_0 + \cdots + \alpha_d = n$ and $x_0 := 1 - x_1 - \cdots - x_d$, and *their related kernel functions*

$$\mathcal{T}_{n,\omega}(\mathbf{x},\mathbf{y}) = \sum_{|\alpha|=n} \omega_\alpha^{(n)}\ B_\alpha(\mathbf{x})\ B_\alpha(\mathbf{y})\ ,\quad \mathbf{x},\mathbf{y} \in \mathcal{S}^d\ ,$$

for given non-negative weights $\omega_\alpha = \omega_\alpha^{(n)}$. Here, $\mathcal{S}^d$ is the *d-variate standard simplex* given by non-negative vectors $\mathbf{x} \in \mathbb{R}^{d+1}$ subject to the restriction $|\mathbf{x}| = 1$.

We discuss various analytical properties of these kernels including their associated *reproducing kernel Hilbert spaces*. In addition, for a given probability measure ρ on $\mathcal{S}^d$, spectral properties of the associated integral operators

$$(\mathcal{L}_{\rho,n,\omega} f)\,(\mathbf{x}) = \int_{\mathcal{S}^d} \mathcal{T}_{n,\omega}(\mathbf{x},\mathbf{y})\ f(\mathbf{y})\ d\rho(\mathbf{y})\ ,\quad n = 0,1\ldots\ ,$$

will be addressed. For special weights $\omega_\alpha^{(n)}$ these integral operators reduce to the Bernstein-Durrmeyer operators for arbitrary measures ρ, for which uniform convergence has been recently characterized by Elena Berdysheva [1].

References

[1] E. Berdysheva, Uniform convergence of Bernstein-Durrmeyer operators with respect to an arbitrary measure, Preprint, 2011.

[2] E. Berdysheva and K. Jetter, Multivariate Bernstein-Durrmeyer operators with arbitrary weight functions, *J. Approximation Theory* **162** (2010), 576-598.

[3] E. Berdysheva, K. Jetter and J. Stöckler, Durrmeyer operators and their natural quasi-interpolants, in: *Topics in Multivariate Approximation and Interpolation* (K. Jetter et al., eds.), pp.1-21, Elsevier, Amsterdam, 2006.

[4] K. Jetter and J. Stöckler, An identity for multivariate Bernstein polynomials, *Comput. Aided Geom. Design* **20** (2003), 563-577.