Kernel Interpolation Beyond the Native Space - A Statistician's Perspective

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Abstract

The standard working assumption in kernel interpolation is that the function f that is to be interpolated belongs to some (smooth) Hilbert space of real-valued functions on $\Omega \subset \mathbb{R}^d$. Under this assumption, kernel interpolants turn out to be optimal in several ways, and error bounds can be derived. In some applications, however, the function of interest is very rough, and moreover, the modeler has no control over the sampling. In these cases a stochastic model for f may be more suitable, and we present one such approach that turns out to be closely related to kernel interpolation. Indeed, even the power function, used in the deterministic setup to characterize the magnitude of the approximation error, reappears in the stochastic framework and quantifies the approximation error, now in stochastic terms. We point out this connection, and we finally compare data-driven methods for selecting a suitable kernel in a numerical study, trying to work out if these methods also allow for a transition from deterministic to stochastic modeling approaches and vice versa.