Coupled Mode Equations and Gap Solitons for the 2D Gross-Pitaevsky equation

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Abstract

Gap solitons near a band edge of a spatially periodic nonlinear PDE can be formally approximated by solutions of Coupled Mode Equations (CMEs). Here we study this approximation for the case of the 2D Periodic Nonlinear Schrödinger / Gross-Pitaevskii Equation with a non-separable potential of finite contrast. Using Lyapunov-Schmidt reduction we give a rigorous justification of the CMEs as an asymptotic model for reversible non-degenerate gap solitons and provide H^s estimates for this approximation. The results are confirmed by numerical examples including some new families of CMEs and gap solitons absent for separable potentials.

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