Deconvolution methods for ultrasonic NDT

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Abstract Ultrasonic testing is the method for volume nondestructive testing for industrial materials like steel. It is long established and widely used. However, as the demands on parts integrity increase in various industrial applications the demand on measurement sensitivity is lifted correspondingly. E.g. the energy sector needs more and more high pressure tubing, which in turn leads to higher demands in terms of integrity. In order to increase sensitivity of any measurement system it is necessary to remove unwanted noise from the signals. This would lead to better signal to noise ratio and thus allows higher amplification of signals or lower assessment thresholds. Although there are many denoising techniques available (e.g. wavelet denoising, see [1, 10], anisotropic diffusion [14], variational methods [11] or hybrid techniques [5]), the specific problems and the high noise level require adapted denoising and detection methods.

In this work we present two deconvolution methods that are both adapted to the applied special ultrasonic pulses. These two methods can be applied in a preprocessing step in order to solve the inverse problem of flaw detection as well as for pure denoising purposes or for efficient storing of the measured ultrasonic data.

The first method is a special matching pursuit (MP) algorithm in order to deconvolve the mixed data (signal and noise), and thus to remove the unwanted noise. The second method is based on the approximate Prony method (APM). Both methods employ some prior knowledge about the measured ultrasonic signal. We assume here that the impulse response of the ultrasonic transducer can be estimated or computed in advance. If the impulse response can only be estimated, we offer an iteration method for adjusting the parameters in the used Gaussian echo model. The MP algorithm is
used to derive a sparse representation of the measured data by a deconvolution and subtraction scheme. An orthogonal variant of the algorithm (OMP) is presented as well. The APM technique also relies on the assumption that the desired signals are sparse linear combinations of (reflections of) the transmitted pulse.

Several test results show that the methods work well even for high noise levels. Further, an outlook for possible applications of these deconvolution methods is given.

**Keywords** time of flight diffraction · matching pursuit · orthogonal matching pursuit · annihilating filters · approximate Prony method · sparse deconvolution · parameter estimating · sparse representation

1 Introduction

Ultrasonic nondestructive testing methods have a long history as reliable techniques for nondestructive testing, e.g. in the fields of weld inspection, for flaw detection in various orientations or for geometry measurements, like wall thickness evaluation. Typically an ultrasonic pulse is transmitted into the material by use of piezo-ceramic elements. The transient pulse will then travel through the material under test and eventually will be reflected by any material inhomogeneity or geometrical obstacle, like a back wall. The reflected signal is recorded using the same piezo-ceramic element, now used as a receiver, and the electrical signal can be visualized. This signal is commonly named an A-scan. Its use as inspection tool brings into light the challenge of developing fast and reliable data processing methods in order to be able to characterize flaws in the material. In particular, the determination of precise positions and dimensions of flaws is a complicated task due to the huge amount of data.

Many ultrasonic testing applications are based on the estimation of the time of arrival (TOA), time of flight diffraction (TOFD) or the time difference of arrival (TDOA) of ultrasonic echos. In order to analyze the received signals, one can usually suppose that the diffracted and backscattered echo from an isolated defect is a time-shifted, frequency-dissipated replica of the transmitted pulse with attenuated energy and inverted phase. In case of various flaw defects, the backscattered ultrasonic signal is a convolution of a modification of the transmitted pulse with the reflection centers. Generally, we are faced with noisy measurements caused by reflections on microstructures of the tested material and electronic disturbances.

Most methods in the literature for estimating of scatterer locations in ultrasonic testing applications are based on cross-correlation methods. Usually, the cross-correlation is computed by shifting the reference echo and integrating over the received signal. For isolated scatterers, the procedure is assumed to produce a peak at the time of arrival of the received signal. This method works well if the reference signal is known and if the noise in the received signal behaves like low level white noise. However, the transmitters impulse response is often not exactly known and the noise level is rather high.

In this paper, we want to introduce two different methods for flaw detection based on sparse deconvolution of ultrasonic signals. For that purpose, we apply a Gaussian echo model for simulating the modified transmitted pulse and solve an optimization problem for simultaneous deconvolution and denoising of the measured signals. If the
impulse response is not exactly known, we propose to apply a method for adjusting
the parameters of the echo model during the iteration process.

For the deconvolution step we provide two different methods; the first method is
based on a (modified) matching pursuit (MP) algorithm, the second uses an annihilat-
ing filter method, also known as approximate Prony method (APM). For the iterative
improvement of model parameters, we employ a Newton approach.

Experimental data discussed in this publication is obtained using standard ultra-
sonic non-destructive testing devices. We follow the standard naming that one mea-
surement is called an A-scan and subsequent A-scans which form a matrix whose
columns are the single A-scans is called a B-scan.

The obtained deconvolution results are sparse vectors that contain only the most
significant information of the original A-scans. In this way, a simple detection of flaw
positions is possible, e.g. by employing a suitable classification method. Moreover,
the proposed techniques allow for efficient storing of A-scans as well as for denoising.
In the latter case, we just convolve the obtained sparse vectors with the ultrasonic
pulse echo.

For our special applications for inspection of weld defects using the TOFD method,
the proposed methods can be further improved by comparison of neighboring A-scans
in order to achieve higher robustness and precision.

2 The model for signal representation

For representation of a received signal \( s(t) \), we suppose that it can be obtained as a lin-
er combination of time-shifted, energy-attenuated versions of the transmitted pulse
function with inverted phase, where each shift is caused by an isolated flaw scattering
the transmitted pulse. Usually, we have only a certain estimate of the transmitted
pulse function. Using the approach in [3], we model the pulse echo by a real-valued
Gabor function of the form

\[
 f_{\theta}(t) = K_\theta e^{-\alpha t^2} \cos(\omega t + \phi),
\]

with the parameters \( \theta = (\alpha, \omega, \phi) \). Here, \( \alpha \) describes the bandwidth factor, \( \omega \) is the
center frequency, and \( \phi \) the phase of the pulse echo. Because of its Gaussian shape
envelope, this model is called Gaussian echo model. The normalization factor \( K_\theta \) is
taken such that \( \| f_{\theta} \|_2 = 1 \). More precisely, we obtain

\[
 K_\theta^{-2} = \| e^{-\alpha t^2} \cos(\omega t + \phi) \|_2^2 = \int_{-\infty}^{\infty} e^{-2\alpha t^2} \cos^2(\omega t + \phi) \, dt \\
 = \frac{1}{2} \int_{-\infty}^{\infty} e^{-2\alpha t^2} (1 + \cos(2\omega t + 2\phi)) \, dt \\
 = \frac{1}{2} \int_{-\infty}^{\infty} e^{-2\alpha t^2} dt + \frac{\cos(2\phi)}{2} \int_{-\infty}^{\infty} e^{-2\alpha t^2} \cos(2\omega t) \, dt \\
 = \frac{\sqrt{\pi}}{2\sqrt{2\alpha}} (1 + \cos(2\phi) e^{-\omega^2/8\alpha}),
\]
Fig. 1 Example for a pulse echo $f_\theta(t)$ (left), 5 amplitudes $\tilde{a}(m)$ (middle), and the superposition $s(t)$.

where we have used that $\int_{-\infty}^{\infty} e^{-2\alpha t^2} \sin(2\omega t) dt = 0$ since the integrand is an odd function. We usually expect that there exists only a small number $M$ of relevant scatterers corresponding to serious flaws in the material while microstructures in the material cause noise. Then the backscattered signal can be approximated by a superposition of time-shifted pulse echos

$$s(t) = \sum_{m=1}^{M} \tilde{a}(m) f_\theta(t - \tau_m) + \nu(t),$$

(1)

where the time shifts $\tau_m$ are related to the location of the relevant flaws, $\tilde{a}(m)$ are the amplitudes, and $\nu(t)$ denotes additive noise. In practice, the number $M$ of pulse functions in (1) is unknown, but we may easily fix an upper bound depending on the application. We are now faced with the inverse problem of finding the relevant time-shifts $\tau_m$ (and the corresponding amplitudes $\tilde{a}_m$) from the given signal $s(t)$, and using an estimate for the parameters $\theta$ that needs to be improved during the computation process. In this paper, we will provide two different procedures for simultaneous deconvolution and denoising of the signal.

Assuming that we have a first estimate of the parameter vector $\theta = (\alpha, \omega, \phi)$, we would like to improve the pulse echos simultaneously. Let $s$ be the measured backscattered signal, and let $F$ be a nonlinear operator that maps the parameter set $(\tilde{a}, \tau, \theta)$ to the function $\sum_{m=1}^{M} \tilde{a}(m) f_\theta(\cdot - \tau_m)$ with $\tilde{a} = (\tilde{a}(1), \ldots, \tilde{a}(M))^T \in \mathbb{R}^M$ and $\tau = (\tau_1, \ldots, \tau_M)^T \in \mathbb{R}^M$. Then, we aim to solve the optimization problem

$$\min_{\tilde{a}, \tau, \theta} \|F(\tilde{a}, \tau, \theta) - s\|_2$$

(2)

under the restriction that the number $M$ of terms in $F(\tilde{a}, \tau, \theta)$ is as small as possible.

We are especially interested in TOFD ultrasonic testing of weld defects. Here, we use two different arrangements. In the first arrangement (inspection of weld seams) we use two probes, one transmitter and one receiver, see Figure 2. The transmitter produces a relatively wide beam spread to maximize the extent of the scan. The two probes are aligned geometrically on each side of the weld, and an A-scan is taken at sequential positions along the length of the seam. A typical A-scan usually detects

1. the lateral signal which travels along the surface of the component and has shortest arrival time;
2. the back wall echo, which has longest transit time, see Figure 3.
In the second arrangement (inspection of back wall deformations), transmitter and receiver coincide and the beam is focussed to the back wall. In case of weld defects, also the corresponding signal reflection can be observed in the A-scan.

We will apply the above model (1) for the analysis of the obtained A-scans. One special difficulty in the inspection of defects for this problem is that the lateral signal and the back wall echo strongly dominate the received signal and therefore hamper the detection of other scatterers.

3 Deconvolution based on greedy algorithms

We are especially interested in real time algorithms for detection of arrival times in the proposed models. Therefore, we propose first a matching pursuit approach. The matching pursuit algorithm has been introduced by Mallat and Zhang [6], see also Tropp [13] and references therein. It has been considered earlier in ultrasonic nondestructive testing; we refer to [12] as well as to modified versions as high resolution pursuit [9] and support matching pursuit [7]. Generally, the matching pursuit algorithm works as follows. Let us assume that a given function $s$ in a Hilbert space $H$ can be well approximated by a linear combination of given functions $b_j$ from a dictionary $\mathcal{D} = \{b_1, \ldots, b_D\}$. In the first step, one iteratively seeks for the dictionary function $b_j$ that correlates best with $s$. Then the same procedure is applied to the residuum $r_1 = s - \langle s, b_j \rangle b_j$ and so forth. In order to apply this idea to our model, we first need a suitable discretization. We suppose in this section that the parameter vec-
tor \( \theta \) describing the pulse functions \( f_\theta \) is given, such that the optimization problem (2) needs (only) to be solved for unknown \( \tau \) and \( \tilde{a} \). A procedure for iterative adjusting of the parameter vector \( \theta \) will be presented in Section 5.

### 3.1 Discretization of the model

In practice, the received signal (A-scan) \( s \) is given as a vector of sampled signal values \( s = (s(n \Delta_t))_{n=0}^{N} \), where \( \Delta_t \) denotes the sampling distance and \( N+1 \) is the number of data.

Further, we can discretize the pulse echo \( f_\theta \) with a sampling distance \( \Delta_v \), i.e. let \( f_\theta = (f_\theta(t \Delta_v))_{t=-L}^{L} \), where we use only a finite number of function values, since \( f_\theta \) decays rapidly. Then a discretization of the received signal \( s \) can be modeled by

\[
s(n \Delta_t) = \sum_{k=0}^{K} a(k) f_\theta(n \Delta_t - k \Delta_v) + \nu(n \Delta_t), \quad n = 0, \ldots, N, \tag{3}
\]

where \( a = (a(k))_{k=0}^{K} \) denotes the vector of (unknown) amplitudes. A comparison of this representation of \( s \) with the sparse representation in (1) yields that we can suppose that only a small number \( M \ll K \) of coefficients in \( a = (a(k))_{k=0}^{K} \) has a modulus being significantly different from zero, i.e., \( a \) contains only \( M \) nonzero components \( a(k_m) = \tilde{a}(m), m = 1, \ldots, M \), and the indices \( k_m \in \mathbb{Z} \) satisfy \( 0 \leq k_1 < \ldots < k_M \leq K \). Hence, the relevant time-shifts \( \tau_m \) in (1) are given by \( \tau_m = m \Delta_v \). We assume that all shifted impulse functions \( f_\theta(t - \tau_m) \) are completely recorded by the sampled data, i.e., \( L \leq k_1 \).

We denote the coefficient matrix of the linear system in (3) by \( F_\theta = F_{\theta, \Delta_v, \Delta_t} = (f_\theta(n \Delta_t - k \Delta_v))_{n=0}^{N},(k=0}^{K} \) and can shortly write

\[
F_{\theta, \Delta_v, \Delta_t} \cdot a + \nu = s, \tag{4}
\]

where \( \nu = (\nu(0), \nu(\Delta_v), \ldots, \nu(N \Delta_v))^T \) is the noise vector. In case of \( \Delta_t = \Delta_v \) we especially obtain the matrix-vector representation

\[
\begin{pmatrix}
f_\theta(-L \Delta_t) & 0 & \cdots & 0 \\
f_\theta((-L + 1) \Delta_t) & f_\theta(-L \Delta_t) & \cdots & \\
\vdots & \vdots & \ddots & f_\theta(-L \Delta_t) \\
f_\theta(L \Delta_t) & \cdots & \cdots & f_\theta((-L + 1) \Delta_t) \\
0 & f_\theta(L \Delta_t) & \cdots & \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & f_\theta(L \Delta_t)
\end{pmatrix} a + \nu = s. \tag{5}
\]

This linear system is overdetermined and needs to be solved approximately under the restriction that the coefficient vector \( a \) is sparse, i.e., contains only \( M \ll K + 1 \) elements. Thus, we look for a solution of the optimization problem

\[
\min_a \|F_\theta a - s\|_2
\]
under the restriction that the subnorm \( \|a\|_0 \), i.e. the number of nonzero components in \( a \), is small.

**Remark.** Theoretically, the sampling distances \( \Delta_t \) and \( \Delta_v \) can be taken differently, where \( \Delta_t \) should be a multiple on \( \Delta_v \). However, since \( L\Delta_v \) and \( K\Delta_v \) are constant values and the support of (the discretized) \( f_\theta \) is bounded by \( [-L\Delta_v, L\Delta_v] \), a decrease of \( \Delta_v \) yields an increase of \( L \) and \( K \). This quickly leads to an underdetermined linear system (3) hampering the evaluation of a sparse solution vector \( a \).

### 3.2 Matching Pursuit

Considering the linear system \( F_\theta a + \nu = s \), we denote the columns of the matrix \( F_\theta \) by \( f_0, \ldots, f_K \). Then the system (4) can also be written in the form

\[
s = \sum_{k=0}^{K} a(k) f_k + \nu,
\]

i.e., \( s \) can be approximated by a linear combination of the columns \( f_k \). In a first step, we determine the index \( k_1 \in \{0, \ldots, K\} \) such that the column \( f_k \) correlates most strongly with \( s \), i.e.

\[
k_1 = \arg \max_{k=0, \ldots, K} |\langle s, f_k \rangle|,
\]

where \( \langle s, f_k \rangle = s^T f_k \) is the standard scalar product of the two vectors \( s \) and \( f_k \).

In the next step, we determine the coefficient \( a(k_1) \) such that the Euclidean norm \( \|s - a(k_1) f_{k_1}\|_2 \) is minimal, i.e.

\[
a(k_1) = \frac{\langle s, f_{k_1} \rangle}{\|f_{k_1}\|_2^2},
\]

where \( \|f_{k_1}\|_2 \) denotes the Euclidean norm of \( f_{k_1} \).

Now we consider the residuum \( r_1 = s - a(k_1) f_{k_1} \) and proceed again with the first step, where \( s \) is replaced by \( r_1 \).

Starting with \( r_0 = s \) and with \( a = 0 \), the summarized algorithm works in the \( j \)-th iteration as follows:

1. Determine an optimal index \( k_j \) such that \( f_{k_j} \) correlates most strongly with the residuum \( r_{j-1} \), i.e.

\[
k_j = \arg \max_{k=0, \ldots, K} |\langle r_{j-1}, f_k \rangle|.
\]

2. Update the coefficient \( a(k_j) \) to \( a(k_j) + \langle r_{j-1}, f_k \rangle/\|f_{k_j}\|_2^2 \), where \( \langle r_{j-1}, f_k \rangle/\|f_{k_j}\|_2^2 \) solves the problem \( \min \|r_{j-1} - x f_{k_j}\|_2 \).

Put \( r_j = r_{j-1} - a(k_j) f_{k_j} \).

In our application, we either stop after a fixed number of iterations of the algorithm, i.e., we a priori fix a bound \( M \) for the number of nonzero coefficients in the vector \( a \), or we stop if

\[
\max_{k=0, \ldots, K} |\langle r_{j-1}, f_k \rangle/\|f_k\|_2^2| < \epsilon \text{ for an a priori chosen } \epsilon.
\]
3.3 Orthogonal Matching Pursuit

The orthogonal matching pursuit algorithm works slightly different. While the first step in each iteration stage is the same as before, the OMP adds a least square minimization in the second step, i.e. we use here

2. Update the coefficients \( a(k_1), \ldots, a(k_j) \) such that \( ||s - \sum_{i=0}^{j} a(k_i) f_{k_i}||_2 \) is minimal, and put \( r_j = s - \sum_{i=0}^{j} a(k_i) f_{k_i} \).

The OMP algorithm is more stable than the simple MP algorithm, since the update of amplitudes in each iteration step ensures a better approximation of the signal \( s \).

However, since we are usually interested in a very small number of significant amplitudes, the MP algorithm already provides good results while being less time-consuming.

4 Deconvolution based on the approximate Prony method

The matching pursuit algorithm gives us a fast approximate solution of the optimization problem with sparsity constraint, but we usually need to fix the number of possible pulse reflections \( M \) in advance. Now, we propose a second method, where we can obtain the number of relevant scatterers during the algorithm. Let us consider again our sparsity model (1)

\[
s(t) = \sum_{m=1}^{M} \tilde{a}(m) f_{\theta}(t - \tau_m) + \nu(t),
\]

where we want to optimize over the time shifts \( \tau = (\tau_1, \ldots, \tau_M) \), the amplitudes \( \tilde{a} = (\tilde{a}(1), \ldots, \tilde{a}(M)) \) and the pulse parameters \( \theta \).

As in the last section, we first assume to have a good estimate for the parameter vector \( \theta \) such that we can concentrate on the computation of \( \tau \) and \( \tilde{a} \) from the samples of \( s \). For that purpose, we now adapt the approximate Prony method considered in [8] as follows.

Let the Fourier transform of a function \( f \in L^1(\mathbb{R}) \) be given by

\[
\hat{f}(\xi) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-it\xi} dt.
\]

Applying the Fourier transform to (1), we obtain

\[
\hat{s}(\xi) = \left( \sum_{m=1}^{M} \tilde{a}(m) e^{-i\xi \tau_m} \right) \hat{f}_\theta(\xi) + \hat{\nu}(\xi).
\]

In our case, the real-valued Gabor function \( f_\theta(t) = K_{\theta} e^{-at^2} \cos(\omega t + \phi) \) is the real part of \( g_\theta(t) = K_{\theta} e^{-it^2} e^{i(\omega t + \phi)} = K_{\theta} e^{i\phi} e^{-at^2} e^{i\omega t} \). We obtain the Fourier transform of \( g_\theta \),

\[
\hat{g}_\theta(\xi) = \frac{K_{\theta} e^{i\phi}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-at^2} e^{-i(\xi - \omega) t} dt = \frac{K_{\theta} e^{i\phi}}{\sqrt{2\alpha}} e^{-(\omega - \xi)^2/4\alpha},
\]
and hence
\[ \hat{f}_0(\xi) = \frac{1}{2} (\hat{g}_0(\xi) + \hat{\hat{g}}_0(\xi)) = \frac{K_0}{2\sqrt{2\alpha}} \left( e^{i\phi} e^{-(\omega-\xi)^2/4\alpha} + e^{-i\phi} e^{-(\omega+\xi)^2/4\alpha} \right). \] (6)

Particularly, the function \( \hat{f}_0(\xi) \) possesses only a zero at \( \xi = 0 \) if \( \phi = \frac{(2\pi+1)\pi}{2} \) while \( \hat{f}(\xi) \neq 0 \) for all \( \xi \neq 0 \). Avoiding the case \( \xi = 0 \), we can hence write
\[ \hat{h}(\xi) := \frac{\hat{f}(\xi)}{\hat{f}_0(\xi)} = \sum_{m=1}^{M} \hat{a}(m) e^{-i\xi \tau_m} + \hat{\epsilon}(\xi), \]
where the noise term \( \hat{\epsilon}(\xi) := \hat{\nu}(\xi)/f_0(\xi) \) is assumed to be small.

For given samples \( \hat{h}(k\Delta_\xi) \), (where \( \Delta_\xi \) is a fixed sampling distance) we now aim to compute the frequencies \( \tau_m \in \mathbb{R}_+ \) and the corresponding amplitudes \( \hat{a}(m) \), for \( m = 1, \ldots, M \) separately using the annihilating filter method. For that purpose, we define the annihilating filter polynomial
\[ \Lambda(z) = \prod_{m=1}^{M} (z - e^{-i\Delta_\xi \tau_m}) = \lambda_M z^M + \lambda_{M-1} z^{M-1} + \ldots + \lambda_0 \]
with \( \lambda_M = 1 \) that possesses the exponentials \( e^{-i\Delta_\xi \tau_m} \) with the desired time-shifts \( \tau_m \) as zeros.

In a first step, we will compute the coefficients \( \lambda_k \) of the annihilating polynomial. We observe that for given sample values \( \hat{h}((k+\ell)\Delta_\xi), k = 0, 1, \ldots, \) and \( \ell = 1, 2, \ldots, \), we have
\[
\sum_{k=0}^{M} \lambda_k \hat{h}((k+\ell)\Delta_\xi) = \sum_{k=0}^{M} \lambda_k \sum_{m=1}^{M} \hat{a}(m) e^{-i\tau_m \Delta_\xi (k+\ell)} + \sum_{k=0}^{M} \lambda_k \hat{\epsilon}((k+\ell)\Delta_\xi) \\
\approx \sum_{m=1}^{M} \hat{a}(m) e^{-i\tau_m \Delta_\xi (\ell)} \sum_{k=0}^{M} \lambda_k (e^{-i\tau_m \Delta_\xi})^k \\
= \Lambda(e^{-i\tau_m \Delta_\xi}) \sum_{m=1}^{M} \hat{a}(m) e^{-i\tau_m \Delta_\xi} \ell = 0,
\]
where we have assumed that the noise term \( \sum_{k=0}^{M} \lambda_k \hat{\epsilon}((k+\ell)\Delta_\xi) \) is negligibly small.

Using the above relation for \( \ell = 1, 2, \ldots, M + 1 \), the unknown coefficients \( \lambda_0, \ldots, \lambda_{M-1} \) of \( \Lambda(z) \) can be computed by finding an approximate zero eigenvector of the Hankel matrix
\[
\mathbf{H} = \begin{pmatrix}
\hat{h}(\Delta_\xi) & \hat{h}(2\Delta_\xi) & \cdots & \hat{h}((M+1)\Delta_\xi) \\
\hat{h}(2\Delta_\xi) & \hat{h}(3\Delta_\xi) & \cdots & \hat{h}((M+2)\Delta_\xi) \\
\vdots & \vdots & \ddots & \vdots \\
\hat{h}((M+1)\Delta_\xi) & \hat{h}((M+2)\Delta_\xi) & \cdots & \hat{h}((2M+1)\Delta_\xi)
\end{pmatrix}.
\]

Observe that the number \( M \) of relevant flaws in the backscattered signal is usually unknown. While we had to fix \( M \) in advance using the matching pursuit algorithm,
we can obtain $M$ here from the data as follows. We apply the above eigenvalue problem to a Hankel matrix $H$ of size $N \times N$, where $N$ is an upper bound of $M$. After computation of the approximate zero eigenvector of $H$, we evaluate the corresponding zeros of the annihilating polynomial $\Lambda(z)$. The zeros of $\Lambda$ that are relevant to us, lie (approximately) on the unit circle, such that we are able to compute its number $M$ and the time shifts $\tau_m$, $m = 1, \ldots, M$.

In the second part of the procedure, we can compute the amplitudes $\tilde{a}_m$ as least square solution of the overdetermined linear system

$$\sum_{m=1}^{M} \tilde{a}_m f_\theta(\ell \Delta t - \tau_m) = s(\ell \Delta t), \quad \ell = 0, \ldots, N,$$

thereby neglecting the noise function $v(t)$.

For application of the first step of above procedure, we need to evaluate the Fourier transform $\hat{h} = \hat{s} / \hat{f}_\theta$ at suitable values $k \Delta \xi$. For this purpose we employ the fast Fourier transform as follows. Assume that we have given the sampled values of the backscattered signal $s = (s(\ell \Delta t))_{\ell=0}^N$. Using linear splines, $s$ can be approximated by the sum

$$\tilde{s}(t) = \sum_{\ell=0}^{N} s(\ell \Delta t) N_2(t - \ell \Delta t),$$

where the B-spline $N_2$ has the support $[-(\Delta t)^{-1}, (\Delta t)^{-1}]$ and is given by

$$N_2(t) = (1 - \Delta t|t|) \text{sinc}\left(\frac{t}{2 \Delta t}\right)^2$$

and with the function $\hat{f}_\theta(\xi)$ that is explicitly given in (6), we obtain the approximate values

$$\hat{h}(\frac{2\pi k}{\Delta t}) = \hat{h}(k \Delta \xi) = \frac{\hat{s}(k \Delta \xi)}{f(\frac{2\pi k}{\Delta t})},$$

where $\Delta \xi := \frac{2\pi}{\Delta t}$, $k = 0, \ldots, N$, and where

$$\sum_{\ell=0}^{N} s(\ell \Delta t) e^{-i \xi_k \Delta \ell}, \quad k = 0, \ldots, N$$

is computed for $\xi_k = \frac{2\pi k}{\Delta t}$ by the fast Fourier transform.

5 Optimization of the parameters

In the preceding sections we have assumed that a reliable estimate of the parameter vector $\theta = \theta^{(0)}$ determining the pulse echo is given. Now we want to propose an alternating minimization procedure for the stepwise improvement of the pulse echo
parameters during the computation process. This alternating minimization algorithm consists of two steps at each iteration level.

In the first step of the $k$th iteration, we use the estimate $\theta^{(k-1)}$ for the parameter vector and solve the minimization problem

$$
(\tilde{a}^{(k)}, \tau^{(k)}) := \arg \min_{\tilde{a}, \tau} \| F(\tilde{a}, \tau, \theta^{(k-1)}) - s \|_2
$$

under the restriction that the number $M$ of terms in $F(\tilde{a}, \tau, \theta^{(k-1)})$ is as small as possible. Here, $F$ denotes the nonlinear operator that maps the parameter set $(\tilde{a}, \tau, \theta)$ to the function $\sum_{m=1}^{M} \tilde{a}(m) f_\theta(\cdot - \tau_m)$, see Section 2.

In the second step of the $k$th iteration, we use the computed vectors $\tilde{a}^{(k)}$ and $\tau^{(k)}$ and solve the minimization problem

$$
\theta^{(k)} := \arg \min_{\theta} \| F(\tilde{a}^{(k)}, \tau^{(k)}, \theta) - s \|_2.
$$

For the first minimization we can apply either the matching pursuit approach considered in Section 3 or the approximate Prony method proposed in Section 4.

For solving the second optimization problem, we now apply the Newton method. While the operator $F(\tilde{a}^{(k)}, \tau^{(k)}, \theta)$ is differentiable with respect to all components $\alpha, \omega, \phi$ of the parameter vector $\theta$, the attempt of direct computation leads to highly nonlinear equations. Therefore we prefer the iterative Newton method.

In our case, the Newton algorithm works as follows. A linearization of the operator around an initial guess $\theta^{(k-1)}$ yields

$$
F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)} + d\theta) \approx D_\theta F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)}) d\theta + F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)}).
$$

Here, $D_\theta F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)})$ denotes the Jacobian of $F$ at $\theta^{(k-1)}$, i.e.,

$$
D_\theta F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)}) = \left( \frac{\partial F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)})}{\partial \alpha}, \frac{\partial F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)})}{\partial \omega}, \frac{\partial F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)})}{\partial \phi} \right).
$$

The update vector $d\theta = (d\alpha, d\omega, d\phi)^T$ can now be obtained from the equation

$$
D_\theta F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)}) d\theta + F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)}) = s
$$

that can be evaluated at the known samples $k\Delta$. This leads to a least squares problem which can be directly solved since the corresponding coefficient matrix has only three dimensions. In this way, we obtain the new update $\theta^{(k-1)} := \theta^{(k-1)} + d\theta$. We proceed with the Newton iteration to obtain the updates $\theta_2^{(k-1)}, \theta_3^{(k-1)}, \ldots$. After $r$ Newton steps that can be just fixed or can depend on some suitable error criterion, we obtain the new estimate $\theta^{(k)} = \theta^{(k-1)}$. For the numerical application of this procedure for parameter optimization we refer to Section 6.
6 Test results

We have tested the proposed procedures for denoising of TOFD data as well as for testing of back wall deformations.

Considering real data, there are mainly two components of noise: a) structural noise, produced by multiple reflections and inhomogeneous material, and b) electronic noise, fed from cables, amplifiers etc., which act like a bandpass filter. The latter can be considered as Gaussian noise in $a$, which results in colored noise after convolution with the wave. Hence, to test the algorithms with different noise levels and different wave forms we have modeled an back wall deformation and added Gaussian noise to $a$ before convolving with the wave.

As in practical applications, the ultrasonic wave send out by the emitter is not given, we estimate it from the given data. For this purpose, we compute (componentwisely) the mean value of all A-scans, i.e., in the matrix of data values (B-scan), where each column represents an A-scan, we compute the mean value of each row separately. In the obtained mean value vector, we separate the back wall echo, normalize its maximal amplitude to 1 and take this result as an approximation of the pulse echo, see Figure 4. The idea behind this procedure is that the mean value procedure gives a good estimate for an A-scan that is obtained by a back wall echo only, since the material flaws are rare and yield signals with a small amplitude.

Figure 5 shows the behavior of OMP (and MP) for a single A-scan. It takes the noisy A-scan and computes its largest amplitudes with a given wave function. By convolving these amplitudes with the wave function we obtain a denoised approximation of the given A-scan.

In Figures 6 – 9, a comparison between reconstructed back walls with different Gabor waves is shown, where always the same initial back wall deformation is used. Each back wall image is again overlaid with noise of different deviation levels and for each Figure the coloring is readjusted. Comparing the modeled examples in these Figures, it seems that the reconstructed back wall echo in $a$ occurs a bit “earlier” than in the noisy image $s$ (the timeruns from top to bottom). The main reason for this fact is that the significant coefficient obtained by the matching pursuit algorithm refers to the starting point of the (approximated) pulse function and not to its maximal amplitude.

![Fig. 4 Vector of mean values of each row and the separated back wall echo as wave approximation: left: nearly nonnegativ wave, noise variance 0.1; centre: symmetric wave, variance 0.025, right: antisymmetric wave, variance 0.001](image-url)
Furthermore, since $F$ is not a square matrix, these two plotted images do not have the same number of rows.

The reconstructed back wall echo time yields the wall thickness of the modeled tube correctly up to the discretization error.

In Figure 6, we present a modeled back wall echo with added noise (left) using different noise levels, the amplitudes of significant translates/reflections of the pulse echo computed with the matching pursuit (MP) method (middle), and with the approximate Prony method (APM) (right). In this case a nearly nonnegative Gabor wave is used as pulse function.

In Figure 7, a similar experiment is applied with a symmetric Gabor wave as pulse echo, using the orthogonal matching pursuit (OMP) algorithm and the APM method. Further, Figure 8 shows the denoising results taking an antisymmetric Gabor wave as pulse echo and using OMP and APM.

We observe, that the MP and the OMP give reasonable results even for highly noisy data. The APM works accurately for the low-level noise case. The reason for that behavior is, that MP/OMP are rather robust algorithms whereas the APM is less numerically stable for high noise levels. Thus the MP/OMP methods are more suitable for a fast determination of material defects while the APM is able to identify clustered defects in the low-level noise case, and may be especially appropriate for determining the more exact structure of a defect, after knowing where that defect is located. This problem will be considered further in the future.

In Figure 9, we consider real data of a TOFD signal and apply a modified MP-method for deconvolution. For TOFD signals the lateral signal as well as the back wall echo have generally significantly larger amplitudes than the signals indicating defects. Similarly as for testing of wall thickness, the reflection obtained from the undamaged back wall is stronger than the relevant signals indicating back wall deformations. In order to obtain the essential signals indicating weld deformations, we add suitable weights that can be chosen a priori using knowledge about the thickness of the tube and an estimate about positions of lateral signal and back wall echo in the A-scan.
This simple adaptation of the MP-method already gives very good results for denoising of A-scans and detection of positions of defects in real data, see Figure 9.

To apply the parameter optimization discussed in Section 5 numerically, we will slightly adjust the Newton method. Because of the complicated normalization factor $K_\theta$ the vector $D_\theta F(\tilde{a}^{(k)}, \tau^{(k)}, \theta^{(k-1)})$ can not easily be computed analytically. Therefore, we consider an analytical representation of

$$D_\theta \left( \frac{1}{K_\theta} F(\tilde{a}, \tau, \theta) \right) = D_\theta \left( \sum_{m=1}^{M} \tilde{a}(m) e^{-\alpha(t-\tau_m)^2} \cos(\omega(t-\tau_m) + \phi) \right)$$
and obtain for $\theta = (\alpha, \omega, \phi)^T$,

$$D_\theta \left( \frac{1}{K_\theta} F(\tilde{a}, \tau, \theta) \right) = 
\begin{pmatrix}
- \sum_{m=1}^{M} \tilde{a}(m)(t - \tau_m)e^{-\alpha(t - \tau_m)^2}\cos(\omega(t - \tau_m) + \phi) \\
- \sum_{m=1}^{M} \tilde{a}(m)(t - \tau_m)e^{-\alpha(t - \tau_m)^2}\sin(\omega(t - \tau_m) + \phi) \\
- \sum_{m=1}^{M} \tilde{a}(m)e^{-\alpha(t - \tau_m)^2}\sin(\omega(t - \tau_m) + \phi)
\end{pmatrix}^T.
$$

However, a change of the parameter vector $\theta$ implies a possibly considerable change of the norm of the wave function $f_\theta$. A disregard of the normalization factor $K_\theta$ thus
leaves to a highly unstable method since the amplitudes in $\tilde{a}$ are optimized with respect to the Euclidean norm of $f_\theta$. In order to counter this problem we are updating not only $\theta$ in each Newton step but also the amplitudes $\tilde{a}$. In this way the amplitudes in $\tilde{a}$ are
Fig. 9  Top: original TOFD data, second row: approximative solution of $a$ with MP  
third row: nonzero elements of the solution, button: approximation of $s \approx F \cdot a$
adjusted to the changing wave norm. We now employ

\[
D_{\theta, \beta} \left( \frac{1}{K_\phi} F(\tilde{a}, \tau, \theta) \right) = \begin{pmatrix} 
- \sum_{m=1}^{M} \tilde{a}(m)(t - \tau_m)^2 e^{-\alpha(t - \tau_m)^2} \cos(\omega(t - \tau_m) + \phi) \\
- \sum_{m=1}^{M} \tilde{a}(m)(t - \tau_m)e^{-\alpha(t - \tau_m)^2} \sin(\omega(t - \tau_m) + \phi) \\
e^{-\alpha(t - \tau_1)^2} \cos(\omega(t - \tau_1) + \phi) \\
\vdots \\
e^{-\alpha(t - \tau_M)^2} \cos(\omega(t - \tau_M) + \phi)
\end{pmatrix}^T
\]

in the Newton method and obtain the following results.
Figures 10, 11, and 12 show the convergence of the Newton method to obtain the correct wave parameter vector $\theta$, or a good approximation of it. In all experiments, the starting vector $\theta^{(0)} = (\alpha^{(0)}, \omega^{(0)}, \phi^{(0)})^T$ has been chosen quite far away from the correct parameter vector. The numerical results show that acceptable parameter adjustment is already achieved after less than 10 iteration steps. In the experiments for Figure 11 and for Figure 12, we have added Gaussian noise to $a$ before convolving the data. The alternating algorithm still converges but the noise causes some minor errors. In Figure 11, the parameters converge not to the original values (blue line) but to values nearby. The parameters in Figure 12 seem to oscillate nearby the original values. However, the approximation of the wave and the original data is good. When no noise is added the algorithm usually converges to the original values (see Figure 10) and one can get a perfect reconstruction of wave and amplitudes.

7 Conclusions and Outlook

The deconvolution methods presented in this work are supposed to be used as a preprocessing step for further applications. Our long term objective is to derive a method to invert the B-scans (where a B-scan is the image obtained from a series of A-scans as columns, see [2]). We would like to reconstruct the shape of the back wall based on the B-scan image. Usually, such inversion techniques provide better results if the raw data only contains low-level noise, and they tend to be unstable if the raw data is too noisy. Hence, it is important to apply a fast and effective denoising algorithm.
that is capable to preserve the important signal features while removing most of the noise.

In this paper, we have proposed two different deconvolution algorithms that both map an A-scan to a sparse vector that still contains the relevant information on the A-scan in an encoded form. This sparse representation of the A-scan resp. the B-scan can be differently processed:

**Flaw detection.** A comparison of the significant coefficients in the sparse columns of the B-scan (after deconvolution) provides the positions of significant flaws in the material. Respectively, in the case of weld seam inspection, the sparse B-scan can be processed further by a direct inversion method. Alternatively, a representation of the B-scan with only a few coefficients can be used for classification using machine-learning algorithms. The algorithm “learns” the B-scans corresponding to different classes (e.g. for different flaws in the back wall) and afterwards tries to assign the correct class to a new unknown B-scan. In such learning procedures, the algorithms are usually not able to handle full images but only a very limited number of representing attributes. Hence, the nonzero coefficients provided by our deconvolution algorithms will act as a good choice of representing attributes for such machine learning algorithms.

**Denoising.** A convolution of the obtained sparse vectors with the (computed or estimated) pulse echo yields a denoised B-scan. Since the deconvolution algorithms are suitably adapted to the measured signals (by using the transmitted pulse echo), this denoising method outperforms most direct (non-adaptive) denoising methods for images (see e.g. Figure 9).

**Compression.** Another advantage of our proposed algorithm is that the nonzero coefficients provide a strong compression of the B-scan. The whole B-scan is reduced to a small number of most significant coefficients, representing the relevant information. Knowing the shape of the pulse, it is possible to reconstruct the B-scan only with the knowledge of the position of the sparse nonzero coefficients. Apparently, this can be used to reduce the amount of storage significantly.

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**References**