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An Iterative Approach for Integrated Planning in Public Transportation*

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Abstract

Optimization in public transport planning is an important topic of ongoing research. Traditionally, the planning process is separated hierarchically into several stages, e.g. line planning, timetabling and vehicle scheduling. Recently, integrated public transport planning, i.e., optimizing several of the planning stages simultaneously, has gained in importance as this can improve the solution quality immensely. However, since the resulting integrated problems are computationally challenging for close-to real-world instances, heuristic solutions are commonly used. We here introduce a new iterative approach for re-optimizing an existing public transport system. For this, two of the three planning stages line planning, timetabling and vehicle scheduling are fixed while the remaining one is re-optimized. To model the re-optimization, traditional approaches do not suffice and therefore new optimization problems need to be defined. We model these problems and propose solution algorithms for each stage which are theoretically analyzed. Additionally, convergence of the proposed iterative approach is discussed theoretically and computationally tested on a benchmark case study and a close-to real-world data set.

Keywords: Public Transport Planning, Line Planning, Timetabling, Vehicle Scheduling, Iterative Heuristic, Integrated Planning

1 Introduction

With rising population numbers in urban areas the need for transportation rises as well. As public transportation is a very efficient, and - compared to individually traveling by car -

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environmentally friendly mode of transport, its importance is increasing. However, the supply of public transport will only increase if its quality - both from an operator's and a passenger's perspective - is sufficiently high. Mathematical public transport planning aims to ensure this quality at various stages of the planning process. Here, we consider three of the most important and well researched problems of public transport planning: *line planning*, *timetabling* and *vehicle scheduling*.

All three problems are well researched on their own. For an overview on line planning, see [Sch12], literature on timetabling can be found in [LLER11] and [BK09] contains an overview of vehicle scheduling models.

Traditionally, these problems are solved sequentially, as depicted in Figure 1.

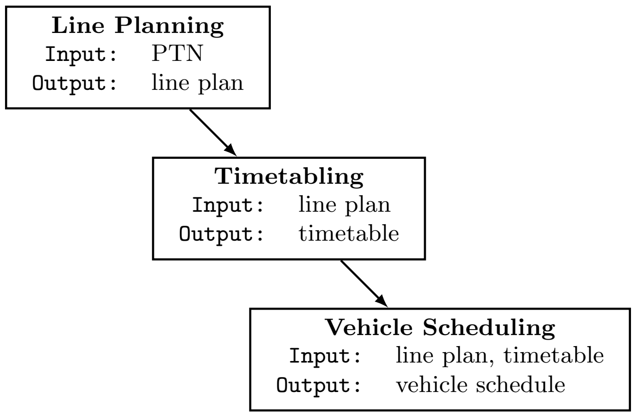


Figure 1: Sequential approach.

However, these problems highly depend on each other as the output of one stage is the input for the next stage. Additionally, we are interested in the overall outcome, i.e., the line plan with corresponding timetable and vehicle schedule which we call a *public transport plan*. Thus, our goal is to solve the following *integrated* problem:

Problem 1 (Public Transport Plan)**.** Find a line plan with a corresponding timetable and vehicle schedule such that the travel time of the passengers and the operational costs are minimized.

Recently, the focus of research concerning public transportation planning has shifted to integrated planning to harvest the benefits of integration.

An important focus is the integration of passenger routing into the single stages, see e.g. [Sch14]. This can be included in the line planning problem ([PB06, SS06, SS15a]) or the timetabling stage ([Sie11, SS15b, GGNS16, BHK17, SS18]). The differences between route assignment which focuses on a system-optimal solution and route choice which models the passengers' behavior more naturally are considered in [GS17].

Another topic of research is the integration of multiple of the three separate stages, i.e., line planning and timetabling, see e.g. [RN09], or timetabling and vehicle scheduling, see e.g. [Lie08, CM12], or even combining all three steps, see e.g. [LPSS18, Sch18].

But as the problems drastically increase in size and thus become even more computationally challenging, heuristic approaches to the integrated problems are more promising. Of course, the traditional sequential approach shown in Figure 1 is such a heuristic but other, more specialized heuristics often perform better.

[BBVL17] developed an iterative approach to line planning and timetabling, solving both steps sequentially. Another approach to the integration of these two problems is the usage of metaheuristics, as done in [TI14]. Both approaches are also applied to the integration of timetabling and vehicle scheduling, see [SE15, FvdHRL18] for a metaheuristic and [GH10, PLM$^+$13] for iterative approaches. Finally, there are also iterative approaches for the integration of all three problems in [MS09] and [PSSS17].

Our Contribution

Here, we present a novel iterative heuristic for the integrated line planning, timetabling and vehicle scheduling problem, attending to the main issue with the sequential approach, i.e., the interdependence of the problems. If a line plan is fixed first and only afterwards a timetable and a vehicle schedule are constructed, this may lead to bad, or even infeasible, solutions, see [GSS13]. Therefore, we develop an iterative approach to re-optimize a given public transport plan where in each step one of the stages is re-optimized and the other ones are regarded as fixed such that a feasible solution is guaranteed, as depicted in Figure 2. For this, two completely new public transportation problems are identified and modeled. An overview can be found in Figure 2. This iterative approach specifies the three steps in the inner circle of the algorithmic scheme called eigenmodel which is introduced in [Sch17].

Of the three algorithms shown in Figure 2, only `ReVehicleScheduling` has been studied before, while `ReLinePlanning` and `ReTimetabling` are newly defined and discussed in Section 3.

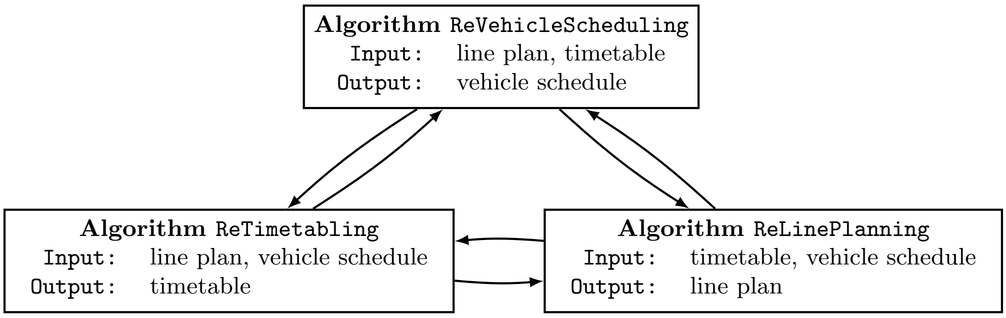


Figure 2: Overview of the algorithms.

Overview of the paper

The remainder of this paper is structured as follows: In Section 2 we formally define a public transport plan by using the classical problems line planning, timetabling and vehicle scheduling. In Section 3 we introduce the models and algorithms for the re-optimization problems where always one of the three stages is re-optimized while the other two stages are fixed. The iterative approach and some theoretical implications are presented in Section 4 while computational experiments on a benchmark data set and close-to real-world data is presented in Section 5.

2 Definition of a Public Transport Plan

In this section, we formally define the parts of a public transport plan, namely line plans, timetables and vehicle schedule, and how to measure its quality.

Note that we consider binary line frequencies in the following which is a common assumption for timetabling, see e.g. [SU89].

We assume the following data to be given. Let PTN=(V, E) be an infrastructure network or *public transport network* with stops or stations V and direct connections E between them. The lower and upper bounds on the wait times at stops are given as L_{wait} and U_{wait} while the lower and upper bounds on the transfer times at stops are given as L_{trans} and U_{trans}. We assume that transfers are always possible, i.e.,

$$U_{\text{trans}} = L_{\text{trans}} + T - 1.$$

For each edge $e \in E$ consider the length len_e and a lower and upper bound L_e and U_e on the drive time on this edge. The passenger demand is given as an OD matrix $C = (C_{u,v})_{u,v \in V}$ where $C_{u,v}$ represents the number of passengers traveling from u to v in the planning period.

The line plan and the timetable are periodic and the length of the planning period is T.

2.1 Line Planning

In the line planning stage, the goal is to cover the edges of the PTN by *lines* chosen from a *line pool* $\mathcal{L}^0$. A *line* is a path in the PTN which has to be covered by a vehicle end-to-end while a *line pool* is a set of lines. The length len_l of a line l is given by the lengths of its edges, i.e.,

$$\mathrm{len}_l = \sum_{e \in l} \mathrm{len}_e.$$

In order to facilitate reasonable travel times for the passengers, *lower frequency bounds* $f_e^{\min}$ have to be satisfied for all edges $e \in E$.

Finding a *line plan* $\mathcal{L}$ amounts to assigning a frequency $f_l \in \{0, 1\}$ to each line $l \in \mathcal{L}^0$. We say a line l is part of line plan $\mathcal{L}$ or $l \in \mathcal{L}$ if $f_l = 1$. A line plan is *feasible* if the following condition is satisfied for all edges $e \in E$:

$$\sum_{\substack{l \in \mathcal{L}: \\ e \in l}} f_l \geq f_e^{\min}.$$

We assume that the lower frequency bounds $f_e^{\min}$, $e \in E$, are given such that the vehicle capacity suffices for routing all passengers in every feasible line plan.

2.2 Timetabling

As we consider periodic timetabling that can be represented by the *periodic event scheduling problem (PESP)* defined in [SU89], we need an *event-activity network (EAN)* $\mathcal{N} = (\mathcal{E}, \mathcal{A})$. For a given line plan $\mathcal{L}$, the EAN consists of a set of events $\mathcal{E}$ which represent the arrival and departure of lines at stops and a set of activities $\mathcal{A}$ representing driving of vehicles on lines, vehicles waiting at stops or passengers transferring at stops.

$$
\begin{aligned}
\mathcal{E} &= \mathcal{E}_{\text{arr}} \cup \mathcal{E}_{\text{dep}} \\
\mathcal{E}_{\text{arr}} &= \{(v, l, \text{arr})\colon v \in l \cap V, l \in \mathcal{L}\} \\
\mathcal{E}_{\text{dep}} &= \{(v, l, \text{dep})\colon v \in l \cap V, l \in \mathcal{L}\} \\
\mathcal{A} &= \mathcal{A}_{\text{drive}} \cup \mathcal{A}_{\text{wait}} \cup \mathcal{A}_{\text{trans}} \\
\mathcal{A}_{\text{drive}} &= \{((v_1, l, \text{dep}), (v_2, l, \text{arr}))\colon \{v_1, v_2\} \in l \cap E, l \in \mathcal{L}\} \\
\mathcal{A}_{\text{wait}} &= \{((v, l, \text{arr}), (v, l, \text{dep}))\colon v \in l \cap V, l \in \mathcal{L}\} \\
\mathcal{A}_{\text{trans}} &= \{((v, l_1, \text{arr}), (v, l_2, \text{dep}))\colon v \in l_1 \cap l_2 \cap V, l_1, l_2 \in \mathcal{L}\}.
\end{aligned}
$$

Each activity $a \in \mathcal{A}$ has a lower and an upper bound L_a and U_a, respectively. Here, the bounds on waiting or transferring at stops are derived from the corresponding PTN bounds $L_{\text{wait}}, U_{\text{wait}}$ and $L_{\text{trans}}, U_{\text{trans}}$, respectively, while the bounds on driving activities $((v_1, l, \text{dep}), (v_2, l, \text{arr}))$ are derived from the corresponding edge $e = \{v_1, v_2\} \in E$ with bounds L_e, U_e.
To find a timetable π, a time point $\pi_i \in \{0, \ldots, T-1\}$ is assigned to each event $i \in \mathcal{E}$. The *duration* $d(a)$ of activity $a = (i, j) \in \mathcal{A}$, is defined as

$$
d(a) = (\pi_j - \pi_i - L_a) \mod T + L_a.
$$

A timetable π is *feasible* if the duration of each activities lies within its lower and upper bounds, i.e., if

$$
L_a \leq d(a) \leq U_a.
$$

To evaluate the quality of a timetable, we assume that the passenger paths are fixed and for each activity $a \in \mathcal{A}$ the number of passengers using it is given as w_a. We call $w = (w_a)_{a \in \mathcal{A}}$ passenger weights. These weights are determined by a routing step ahead of the optimization, e.g. by assigning to each OD pair a shortest path according to the lower bounds on the activities. The goal of the optimization is to minimize the travel time of the passengers, i.e.,

$$
\mathcal{R}_{\text{fix}}(\pi, w) = \sum_{a=(i,j) \in \mathcal{A}} w_a \cdot d(a). \tag{1}
$$

2.3 Vehicle Scheduling

Vehicle scheduling for a fixed line plan and a fixed timetable is a well researched problem, see e.g. [BK09]. There exist many different variants, with or without one or multiple depots, with or without a maximal number of vehicles which can be used and with different objectives.

We here consider a model with an unlimited number of vehicles, without a depot and minimize a weighted sum of the number of vehicles, the time needed and the distance covered.

In contrast to line planning and timetabling where a plan is computed for a relatively short time span and then repeated, vehicle schedules are computed for longer time spans. For example, a timetable might repeat every hour, while the vehicle schedule is computed for the whole day and only repeated the next day. We therefore consider an aperiodic problem where each line l of the line plan is to be covered $p_{\max}$ times by a vehicle and the p-th covering of line l is called *trip* (p, l). A trip (p, l) is determined by the line l it covers, the period repetition p it starts in, its start time $\texttt{start}_{p,l}$ and its end time $\texttt{end}_{p,l}$. The duration of trip (p, l), $\texttt{duration}_{p,l}$, is the time between $\texttt{start}_{p,l}$ and $\texttt{end}_{p,l}$. The length of a trip (p, l), $\text{len}_{p,l}$, is the length of the corresponding line l, i.e., $\text{len}_{p,l} = \text{len}_l$. A *vehicle route* is a list of *compatible* trips where two trips (p_1, l_1), (p_2, l_2) are compatible if there is sufficient time to get from the last station of line l_1 to the first station of line l_2 on a fixed shortest path P, i.e., if

$$\texttt{start}_{p_2,l_2} - \texttt{end}_{p_1,l_1} \geq L_{l_1,l_2},$$

where L_{l_1,l_2} is the needed time to directly drive from the last stop of l_1 to the first stop of l_2 and D_{l_1,l_2} as the corresponding distance. We assume that

$$L_{l_1,l_2} = \sum_{e \in P} L_e$$
$$D_{l_1,l_2} = \sum_{e \in P} \text{len}_e$$

is satisfied. A *vehicle schedule* is a set of vehicle routes such that all trips in $\mathcal{T} = \{(p, l) \colon p \in \{1, \dots, p_{\max}\}, l \in \mathcal{L}\}$ are covered exactly once.

2.4 Objectives

To evaluate the quality of a public transport plan, we consider two objectives, namely the operational costs and the travel time for the passengers.

The *travel time of the passengers* is measured on shortest paths according to the timetable. Note that this may not be the same as the objective function of the timetabling problem as passengers can choose a new, possibly shorter, path. For this, let $P_{u,v}(\pi)$ be a shortest path from any departure event at stop u to any arrival event at stop v w.r.t the timetable π. We therefore measure the rerouted travel time

$$\mathcal{R}_{\mathrm{SP}}(\pi) = \sum_{(u,v)\in C} C_{u,v} \cdot \sum_{a\in P_{u,v}(\pi)} d(a).$$

It is also possible to instead measure the *perceived travel time* where transfers are penalized by a fixed penalty term. This can easily be added to the models presented here by modifying the duration of transfer activities. For easier notation, we consider travel time in the remainder of this paper.

The *operational costs* are determined by the vehicle schedule and include duration based costs $\mathrm{cost}_{\mathrm{time}}$, distance based costs $\mathrm{cost}_{\mathrm{len}}$ and costs per vehicle $\mathrm{cost}_{\mathrm{veh}}$. In addition to the distance and time needed to cover the trips, vehicles also have to relocate between trips. In order to compute the costs of this relocation, we define *connecting trips*. Let $r = ((p_1, l_1), (p_2, l_2), \ldots, (p_n, l_n))$ be a vehicle route. Then for each $i \in \{1, \ldots, n-1\}$ the tuple $((p_i, l_i), (p_{i+1}, l_{i+1}))$ is called a connecting trip. The duration of connecting trip $c_i = ((p_i, l_i), (p_{i+1}, l_{i+1}))$, $\texttt{duration}_{c_i}$, is the time between the end of trip (p_1, l_1) and the start of trip (p_2, l_2) and its length, len_{c_i}, is $D_{l_i,l_{i+1}}$, i.e., the distance to cover when driving from l_i to l_{i+1}.

Let $\mathcal{V} = \{r_1, \ldots, r_n\}$ be a vehicle schedule with vehicle routes r_i. Then the operational costs of $\mathcal{V}$ are

$$
\begin{aligned}
\mathrm{cost}(\mathcal{V}) = &\sum_{r\in\mathcal{V}} \Bigg(\sum_{\substack{\text{trip}\\ t=(p,l)\in r}} \mathrm{cost}_{\mathrm{len}} \cdot \mathrm{len}_t + \mathrm{cost}_{\mathrm{time}} \cdot \texttt{duration}_t \\
&\qquad + \sum_{\substack{\text{connecting trip}\\ c=((p_1,l_1),(p_2,l_2))\in r}} \mathrm{cost}_{\mathrm{len}} \cdot \mathrm{len}_c + \mathrm{cost}_{\mathrm{time}} \cdot \texttt{duration}_c \Bigg) \\
&+ \mathrm{cost}_{\mathrm{veh}} \cdot |\mathcal{V}| \\
= &\sum_{r\in\mathcal{V}} \Bigg(\sum_{\substack{\text{trip}\\ (p,l)\in r}} \mathrm{cost}_{\mathrm{len}} \cdot \mathrm{len}_l + \mathrm{cost}_{\mathrm{time}} \cdot (\texttt{end}_{p,l} - \texttt{start}_{p,l}) \\
&\qquad + \sum_{\substack{\text{connecting trip}\\ ((p_1,l_1),(p_2,l_2))\in r}} \big(\mathrm{cost}_{\mathrm{len}} \cdot D_{l_1,l_2} \\
&\qquad\qquad + \mathrm{cost}_{\mathrm{time}} \cdot (\texttt{start}_{p_2,l_2} - \texttt{end}_{p_1,l_1})\big) \Bigg) \\
&+ \mathrm{cost}_{\mathrm{veh}} \cdot |\mathcal{V}|.
\end{aligned}
$$

3 Modelling the Re-Optimization Problems

In this section, we define the re-optimization problems `ReVehicleScheduling`, `ReTimetabling` and `ReLinePlanning` that we need for the iterative approach. For a given public transport plan, our goal is to always fix the solutions of two of the three stages line planning, timetabling and vehicle scheduling while re-optimizing the third stage.

3.1 Re-Optimizing the Vehicle Schedule

As mentioned in Section 2.3, vehicle scheduling for a fixed line plan and a fixed timetable is part of the classical sequential planning process and a well researched problem. Therefore, we can use a standard vehicle scheduling model for `ReVehicleScheduling`. Here, we use a vehicle scheduling model without depot and we minimize the operational costs as defined in Section 2.4. The algorithm used for the experimental evaluation is implemented in the open source software tool LinTim, see [SAP+18].

Problem 2 (`ReVehicleScheduling`)**.** Given a public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ with line plan $\mathcal{L}$, periodic timetable π and vehicle schedule $\mathcal{V}$ covering $p_{\max}$ period repetitions. Let L_{l_1,l_2}, $l_1, l_2 \in \mathcal{L}$, be the minimal durations of the potential connecting trips and D_{l_1,l_2}, $l_1, l_2 \in \mathcal{L}$, the lengths of the potential connecting trips. Let $(\text{cost}_{\text{time}}, \text{cost}_{\text{len}}, \text{cost}_{\text{veh}})$ be given cost parameters.
Find a new feasible vehicle schedule $\mathcal{V}'$ for timetable π, minimal durations of connecting trips L_{l_1,l_2}, $l_1, l_2 \in \mathcal{L}$, and trips $\mathcal{T} = \{(p, l) \colon p \in \{1, \ldots, p_{\max}\}, l \in \mathcal{L}\}$ such that the operational costs $\text{cost}(\mathcal{V}')$ are minimized.

3.2 Re-Optimizing the Timetable

So far, we only described the standard timetabling problem. As mention in Section 2.2, a timetable which is feasible already adheres to the line plan, as it is part of the input and the structure of the EAN. To achieve that also a given vehicle schedule $\mathcal{V}$ stays feasible after a new timetable is found, we need to add further constraints.
Therefore, we consider the set $\mathcal{C}$ of all connecting trips of vehicle routes in $\mathcal{V}$. Remember that connecting trip $c = ((p_1, l_1), (p_2, l_2)) \in \mathcal{C}$ means that trip (p_2, l_2) is operated directly after trip (p_1, l_1) by the same vehicle. In order to check that the vehicle schedule remains feasible, we need to ensure that the minimal time L_{l_1,l_2} between trips on lines l_1 and l_2 is complied with for all connecting trips $c = ((p_1, l_1), (p_2, l_2)) \in \mathcal{C}$.
An important factor is the distribution of passengers to activities of the event-activity network, especially when the event-activity network is modified during the iteration scheme. Thus the passenger weights $w = (w_a)_{a \in \mathcal{A}}$, have to be determined before applying Algorithm `ReTimetabling` by a passenger routing. We choose to route the OD pairs on shortest paths in the EAN according to the previous timetable which allows for a convergence result later on.

Problem 3 (`ReTimetabling`)**.** Given a public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ with line plan $\mathcal{L}$, periodic timetable π for period length T and bounds L_a, U_a on the activities $a \in \mathcal{A}$ of the corresponding EAN $\mathcal{N} = (\mathcal{E}, \mathcal{A})$ and vehicle schedule $\mathcal{V}$. Let L_{l_1,l_2}, $((p_1, l_1), (p_2, l_2)) \in r$, $r \in \mathcal{V}$, be the minimal durations of the connecting trips. Let $w = (w_a)_{a \in \mathcal{A}}$ be passenger weights corresponding to a passenger routing on shortest paths according to timetable π.
Find a new periodic timetable π' that is feasible corresponding to the minimal and maximal bounds on the activities as well as the minimal times for the connecting trips and minimizes the travel time of the passengers for fixed weights $w = (w_a)_{a \in \mathcal{A}}$.

IP Formulation To give an integer program for the problem `ReTimetabling` we adapt the classical PESP formulation and use the following variables. Let $\pi_i \in \{0, \dots, T-1\}$ be the scheduled periodic time of event $i \in \mathcal{E}$, $z_a \in \mathbb{Z}$ the modulo parameter of activity $a \in \mathcal{A}$ and $\texttt{duration}_l \in \mathbb{N}$ the time it takes in the timetable to get from $\texttt{first}(l)$ to $\texttt{last}(l)$. Here, $\texttt{first}(l)$ is the first event in line l while $\texttt{last}(l)$ is the last event in line l. For easier notation we define variables $\texttt{start}_{p,l} \in \mathbb{N}$ for the start time of trip (p, l) and $\texttt{end}_{p,l} \in \mathbb{N}$ for its end time. Let $\mathcal{A}(l)$ be the activities belonging to line l, i.e., all activities $a = (i, j)$ where both events i and j are departure or arrival events of line l. Then we get the following IP formulation.

$$
\begin{aligned}
(\texttt{ReTimetabling})\ \min \sum_{a=(i,j)\in\mathcal{A}} & w_a \cdot (\pi_j - \pi_i + z_a \cdot T) && && \\
\text{s.t.}\quad \pi_j - \pi_i + z_a \cdot T &\leq U_a && a=(i,j)\in\mathcal{A} && (2)\\
\pi_j - \pi_i + z_a \cdot T &\geq L_a && a=(i,j)\in\mathcal{A} && (3)\\
\texttt{duration}_l &= \sum_{a=(i,j)\in\mathcal{A}(l)} (\pi_j - \pi_i + z_a \cdot T) && l\in\mathcal{L} && (4)\\
\texttt{start}_{p,l} &= p\cdot T + \pi_{\texttt{first}(l)} && (p,l)\colon (\bullet,(p,l))\in\mathcal{C} && (5)\\
\texttt{end}_{p,l} &= p\cdot T + \pi_{\texttt{first}(l)} + \texttt{duration}_l && (p,l)\colon ((p,l),\bullet)\in\mathcal{C} && (6)\\
L_{l_1,l_2} &\leq \texttt{start}_{p_2,l_2} - \texttt{end}_{p_1,l_1} && ((p_1,l_1),(p_2,l_2))\in\mathcal{C} && (7)\\
\pi_i &\in \{0,\dots,T-1\} && i\in\mathcal{E} && \\
z_a &\in \mathbb{Z} && a\in\mathcal{A} && \\
\texttt{duration}_l &\in \mathbb{N} && l\in\mathcal{L} && \\
\texttt{start}_{p,l} &\in \mathbb{N} && (p,l)\colon (\bullet,(p,l))\in\mathcal{C} && \\
\texttt{end}_{p,l} &\in \mathbb{N} && (p,l)\colon ((p,l),\bullet)\in\mathcal{C} &&
\end{aligned}
$$

Constraints (2) and (3) are the standard timetabling constraints while equation (4) determines the time it takes to traverse line $l \in \mathcal{L}$. Equations (5) and (6) determine the actual start and end times of trip $(p,l) \in r$, $r \in \mathcal{V}$, respectively. Note that to determine $\texttt{end}_{p,l}$ it is not sufficient to use the time of $\texttt{last}(l)$ for period repetition p as the duration of the traversal of l can be longer than the period length T, see Example 5. Constraint (7) makes sure that the minimal time for connecting trips is complied with.

Remark 4. The given IP formulation can easily be extended to the integrated timetabling and vehicle scheduling problem, by making the vehicle connecting trips variable and adding corresponding flow constraints which makes the problem substantially larger. For details, see [LPSS18, Sch18].

Example 5 ([Sch18]). Consider two lines l_1, l_2 with $L_{l_1,l_2} = L_{l_2,l_1} = 5$. Let the trip length of l_1 which is determined by the bound of the activities belonging to l_1 be in $[60, 120]$ and the trip length of l_2 be fixed to 50 with a planning period of length 60. A possible timetable is given in

Figure 3.

Depending on the actual duration of line l_1 which might be 60 or 120, we need to implement two different vehicle schedules. If the duration is 60, we can find a vehicle schedule with two vehicles. Vehicle V_1 operates trips (1, l_1), (2, l_2), (3, l_1) etc. and Vehicle V_2 operates trips (1, l_2), (2, l_1), (2, l_2) etc. But if the duration is 120, the vehicle operating (1, l_1) cannot operate (2, l_2) and we need a third vehicle to cover all trips although the periodic difference between $\texttt{last}(l_1)$ and $\texttt{first}(l_2)$ is large enough to accommodate a connecting trip.

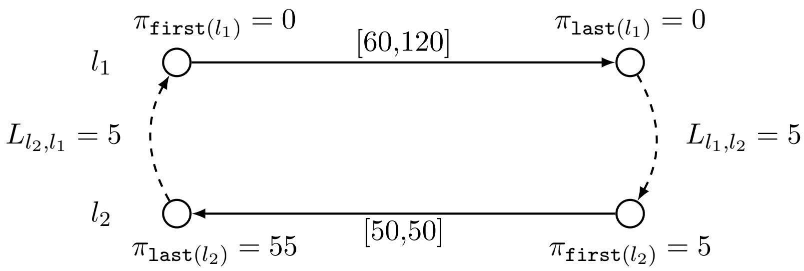


Figure 3: A possible timetable for Example 5.

3.3 Re-Optimizing the Line Plan

For defining the problem `ReLinePlanning`, we first need to understand how to generate new lines that are consistent with the timetable and the vehicle schedule which are already in place. As lines define a physical path that has to be covered by one vehicle end-to-end, they are an integral part of both the vehicle schedule and the timetable. As lines have to appear periodically, we have to make sure that a path can only be a line if it is covered by one vehicle end-to-end in each planning period at the same periodic time. This is especially difficult as we consider the general case of aperiodic vehicle schedules instead of periodic ones as it is done, e.g. in [DRB$^+$17, BKLL18].

For formally defining when lines are consistent with a given timetable and vehicle schedule, let $r = ((p_1, l_1), \ldots, (p_n, l_n))$ be a vehicle route. As every connecting trip between two trips (p_i, l_i), (p_{i+1}, l_{i+1}) is operated on a fixed shortest path, we can determine the physical path of the vehicle, i.e., the path the vehicle takes in the PTN, which we call $P(r)$. For an edge $e \in (p, l)$ with $l = (l', e, l'')$ we determine the aperiodic departure time as

$$\tau_{(e,p,l)} = p \cdot T + \sum_{v \in l' \cap V} \texttt{duration}((v, \text{arr}, l), (v, \text{dep}, l)) + \sum_{(u,v) \in l' \cap E} \texttt{duration}((u, \text{dep}, l), (v, \text{arr}, l)).$$

Note that due to Example 5 we cannot simply compute the aperiodic departure time of e by adding $p \cdot T$ to the periodic departure time of e.

Let $c = ((p_1, l_1), (p_2, l_2))$ be a connecting trip with path $(e_1, \ldots, e_k)$. Note that due to our assumptions this path is a fixed shortest path from the last station of line l_1 to the first station of line l_2. For an edge $e_j \in (e_1, \ldots, e_k)$, we define the departure time as

$$\tau_{(e_j,c)} = p \cdot T + \mathtt{duration}_l + \sum_{i=1}^{j-1} \mathtt{duration}(e_i, c).$$

Here, $\mathtt{duration}(e_i, c)$ is the duration of the edge in the connecting trip, i.e., the time the vehicle takes to cover e_i. These durations have to satisfy

$$\mathtt{duration}(e_i, c) \geq L_{e_i}, \quad i \in \{1, \ldots, k\} \tag{8}$$

$$\sum_{i=1}^{k} \mathtt{duration}(e_i, c) = \mathtt{duration}_c. \tag{9}$$

As changing lines influences the basic level of the corresponding timetable and vehicle schedule, lines cannot even change names without formally changing the timetable and vehicle schedule as lines are used for encoding events and trips. Therefore, we slightly adapt the timetable and the vehicle schedule for a new line plan without changing the physical routes of vehicles during the operation of trips and without changing the times of events that are covered by the new line plan. We thus define consistency of transport plans which are derived from one another by changing the line plan.

Definition 6. Let $(\mathcal{L}, \pi, \mathcal{V})$ be a public transport plan that is feasible according to upper and lower activity bounds derived from the corresponding PTN bounds L_e, U_e, $e \in E$, $L^{\text{wait}}, U^{\text{wait}}$, $L^{\text{trans}}, U^{\text{trans}}$. Let L_{l_1,l_2}, $l_1, l_2 \in \mathcal{L}$ be the minimal durations of the potential connecting trips. A public transport plan $(\mathcal{L}', \pi', \mathcal{V}')$ is *consistent* with $(\mathcal{L}, \pi, \mathcal{V})$, if the following conditions are satisfied.

- $\mathcal{L}'$ is a set of lines with corresponding timetable π' and vehicle schedule $\mathcal{V}'$ which are feasible according to upper and lower activity bounds derived from the corresponding PTN bounds and the minimal times for connecting trips.
- There exists a bijection $b \colon \mathcal{V} \to \mathcal{V}'$.
- For all vehicle routes $r \in \mathcal{V}$ the paths of all trips in $b(r)$ are contained in the path $P(r)$,

i.e., the new vehicle routes cover the same paths as the old vehicle routes when operating trips but might deviate from them for connecting trips. For an edge e contained in trip $(p,l) \in r$ and in a trip $(p',l') \in b(r)$ at the same part of the vehicle route, we denote (p',l') as $b'(e,p,l)$. Analogously, for an edge e contained in connecting trip $c \in r$ and in a trip $(p',l') \in b(r)$ at the same part of the vehicle route, we denote (e,c) as $\bar{b}(e,p',l')$.

- For all edges e contained in a trip (p,l) in vehicle route r and in a trip $b'(e,p,l) = (p',l')$ in vehicle route $b(r)$ the aperiodic departure times coincide, i.e., $\tau_{(e,l,p)} = \tau_{(e,l',p')}$.
- There have to be durations $\texttt{duration}(e,c)$, $e \in c$, $c \in r$, $r \in \mathcal{V}$, according to (8) and (9) such that the following condition is satisfied: Let $(e_1,\dots,e_k) \subset l'$ be the largest subpath of (p',l') in vehicle route $b(r)$ that is completely contained in c. Then the aperiodic departure times $\tau_{(e_i,p',l')}$ satisfy

$$\tau_{(e_k,p',l')} - \tau_{(e_1,p',l')} = \sum_{i=1}^{k} \texttt{duration}(\bar{b}(e_i,p',l')),$$

  i.e., the duration of connecting trip c allows for the operation of line l'.

With this definition, we call a line l *consistent with a public transport plan* $(\mathcal{L},\pi,\mathcal{V})$ if there exists a public transport plan $(\{l\},\pi',\mathcal{V}')$ that is consistent with $(\mathcal{L},\pi,\mathcal{V})$. If a line l is consistent to $(\mathcal{L},\pi,\mathcal{V})$, the following requirements have to be satisfied as direct implications of Definition 6.

- Line l is operated periodically and all corresponding activity durations are feasible as π' is a feasible periodic timetable.
- Line l is covered by one vehicle end-to-end in each planning period as $\mathcal{V}'$ is a feasible vehicle schedule.
- For each trip (p,l), $p \in \{1,\dots,p_{\max}\}$, the path of line l is part of an old vehicle route due to bijection b.
- The departures times at stations that have formerly also been part of a line are the same as before due to the constraints on the aperiodic departure times.
- The duration of the parts of the line that have formerly been connecting trips fit to the duration of the connecting trip.

To ensure a certain service level for the passengers when minimizing the costs of the new line concept, we use the standard line planning constraints, i.e., we consider fixed minimal frequencies on all PTN edges as described in Section 2.1.

As the operational costs do not only depend on the line plan, we approximate them by using costs per line as it is commonly done in line planning, see e.g. [CvDZ98]. We determine the line costs cost_l by using a fixed cost part, a part depending on the length of the edges and one depending on the number of edges, as done e.g. in [GHS17]. The costs of the line plan are therefore

$$\text{cost}(\mathcal{L}) = \sum_{l \in \mathcal{L}} \text{cost}_l. \tag{10}$$

The problem `ReLinePlanning` can now be stated as follows.

> **Problem 7** (`ReLinePlanning`)**.** Given a public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ for PTN (V, E) with line plan $\mathcal{L}$ with minimal edge frequencies $f_e^{\min}$, $e \in E$, duration bounds L_e, U_e, $e \in E$, $L^{\text{wait}}, U^{\text{wait}}$, $L^{\text{trans}}, U^{\text{trans}}$, periodic timetable π for period length T and vehicle schedule $\mathcal{V}$ for $p_{\max}$ period repetitions. Let L_{l_1,l_2}, $l_1, l_2 \in \mathcal{L}$, be the minimal durations of the potential connecting trips.
>
> Find a new public transport plan $(\mathcal{L}', \pi', \mathcal{V}')$ that is consistent with $(\mathcal{L}, \pi, \mathcal{V})$ and minimizes the line costs $\text{cost}(\mathcal{L}')$.

In order to find a new line plan, we first need to create a line pool consisting of lines that are consistent with the original public transport plan. In a second step, we chose a line plan from this pool that can be extended to a public transport plan consistent with the original one. Both steps are described in Algorithm 1.

Algorithm 1 `ReLinePlanning`

Input: PTN$=(V,E)$, lower frequency bounds $f_e^{\min}$, $e \in E$, lower and upper duration bounds L_e, U_e, $e \in E$, $L^{\text{wait}}, U^{\text{wait}}$, $L^{\text{trans}}, U^{\text{trans}}$, period length T, number of period repetitions $p_{\max}$, minimal times for potential empty trips L_{l_1,l_2}, $l_1, l_2 \in \mathcal{L}$, public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ with $\mathcal{V} = \{r_1, \dots, r_n\}$ and vehicle V_i operating route r_i.
Output: A public transport plan $(\mathcal{L}', \pi', \mathcal{V}')$ consistent to $(\mathcal{L}, \pi, \mathcal{V})$.
$\triangleright$ Define line network.
Initialize line network $L = (V_L, E_L)$ with $V_L = V$, $E_L = \emptyset$.
for route $r_i \in \mathcal{V}$ **do**
    for trip edges $e \in (p,l)$, $(p,l) \in r_i$ **do**
        $\triangleright$ Add edge e labeled by aperiodic departure time and vehicle.
        $E_L = E_L \cup \{(e, \tau_{(e,p,l)}, V_i)\}$
    end for
    Fix durations `duration`(e,c), $e \in c$, $c \in r_i$ satisfying (8) and (9).
    for connecting trip edges $e_j \in c$, $c \in r_i$ with $c = (e_1, \dots, e_k)$, $e_j = (u,v)$ **do**
        $\triangleright$ Add edge e_j labeled by aperiodic departure time
        $\triangleright$ and vehicle id if it can be used by passengers.
        if $\tau_{(e_{j+1},c)} - \tau_{(e_j,c)} \in [L_{e_j} + L^{\text{wait}}, U_{e_j} + U^{\text{wait}}]$ **then**
            $E_L = E_L \cup \{(e_j, \tau_{(e_j,c)}, V_i)\}$
        end if
    end for
end for

19: ▷ Define collapsed line network

20: Initialize collapsed line network $C = (V_C, E_C)$ with $V_C = V$, $E_C = \emptyset$.

21: **for** $(e, \tau, V_i) \in E_L$ with $\tau \in \{T, \dots, 2 \cdot T - 1\}$ **do**

22: ▷ Combine parallel edges from the line network

23: ▷ with the same periodic departure time.

24: $\quad E_L = E_L \setminus \{(e, \tau, V_i)\}$, VehList=$[V_i]$, $E_{\text{temp}} = \emptyset$.

25: $\quad$ **for** $p = 1, \dots, p_{\max} - 1$ **do**

26: $\quad\quad$ **if** $\exists (e, \tau + p \cdot T, V_k) \in E_L$ **then**

27: $\quad\quad\quad$ VehList=[VehList, V_k], $E_{\text{temp}} = E_{\text{temp}} \cup \{(e, \tau + p \cdot T, V_k)\}$

28: $\quad\quad$ **else**

29: $\quad\quad\quad$ Start next iteration in line 21.

30: $\quad\quad$ **end if**

31: $\quad$ **end for**

32: $\quad E_L = E_L \setminus E_{\text{temp}}$, $E_C = E_C \cup \{(e, \tau \bmod T, \text{VehList})\}$

33: **end for**

34: ▷ Construct line pool.

35: Find set of longest paths $\mathcal{P}$ in collapsed line network C, s.t. all edges in a path have identical labels VehList and the departure times of two consecutive edges $(e_1 = (u, v), \pi_1, \text{VehList}), (e_2 = (v, w), \pi_2, \text{VehList})$ satisfy

$$(\pi_2 - \pi_1 - L_{e_1} - L^{\text{wait}}) \mod T + L_{e_1} + L^{\text{wait}} \in [L_{e_1} + L^{\text{wait}}, U_{e_1} + U^{\text{wait}}].$$

36: Set the line pool $\mathcal{L}^0$ as the set of all subpaths of $\mathcal{P}$.

37: Find a line plan $\mathcal{L}'$ by solving a line planning problem for pool $\mathcal{L}^0$ such that

38: $\quad$ all PTN edges are covered according to the lower frequency bounds $f_e^{\min}$,

39: $\quad$ all edges $e \in E_C$ are part of at most one line in $\mathcal{L}'$

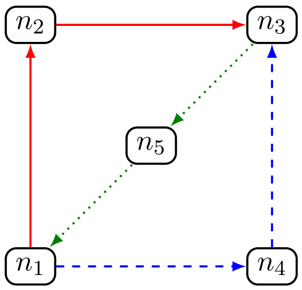
40: $\quad$ and the line costs are minimized.

41: ▷ Find the corresponding timetable and vehicle schedule.

42: Construct timetable π' and vehicle schedule $\mathcal{V}'$ by using the periodic times from the collapsed line network for the departure times, adding the corresponding arrival times and updating the vehicle routes according to the new lines.

The functionality of Algorithm 1 is demonstrated in the following Example 8.

Example 8. We consider the PTN shown in Figure 4, consisting of five nodes and six edges. There are three lines with their corresponding periodic timetable given. The first number stands for the arrival time of the line in the specified station, the second one for the departure time.



Lines

$l_1 = (n_1[00', 05'], n_2[15', 20'], n_3[25', 30'])$

$l_2 = (n_3[30', 35'], n_5[40', 45'], n_1[55', 00'])$

$l_3 = (n_1[00', 05'], n_4[20', 25'], n_3[35', 40'])$

Figure 4: PTN and line plan.

The next figure, Figure 5, shows the vehicle schedule which consists of two vehicle routes. The first vehicle V_1 operates line l_1 and line l_2 alternately while the second vehicle V_2 operates only line l_3.

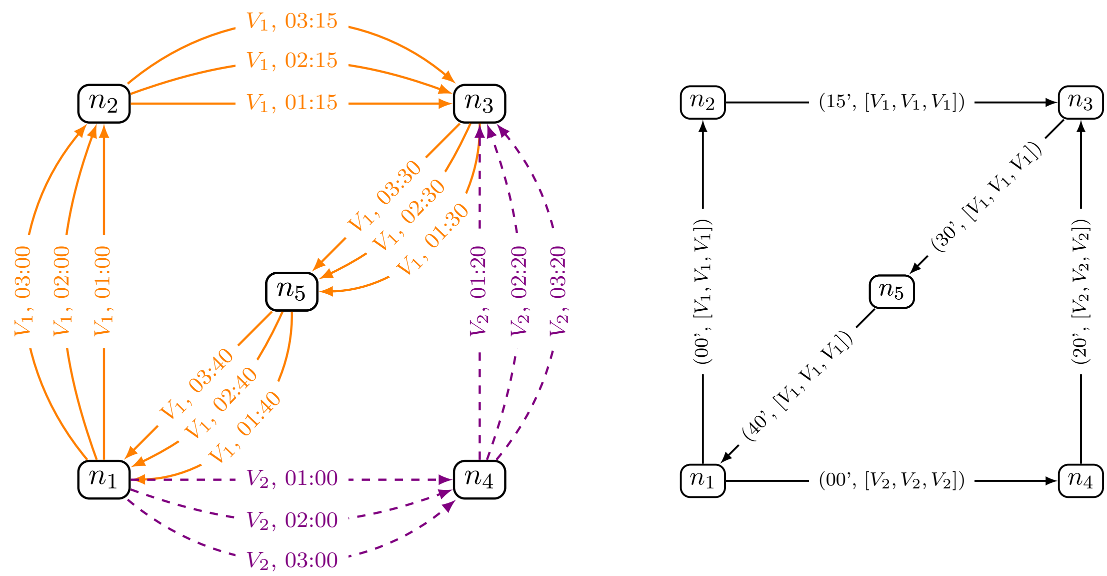
Vehicle V_1:
l_1 [01:00, 01:30], l_2 [01:30, 02:00]
l_1 [02:00, 02:30], l_2 [02:30, 03:00]
l_1 [03:00, 03:30], l_2 [03:30, 04:00]

Vehicle V_2:
l_3 [01:00, 01:40], $\emptyset$ [01:40, 02:00]
l_3 [02:00, 02:40], $\emptyset$ [02:40, 03:00]
l_3 [03:00, 03:40], $\emptyset$ [03:40, 04:00]

Figure 5: Vehicle schedule.

From this information we now create the line network shown in Figure 6a. Here, we see each driving of a PTN edge marked by the vehicle id and the starting time for the three period repetitions we are looking at where the period length is 60 minutes.

The collapsed line network is shown in Figure 6b. Here, the periodic drivings are shown, marked by the periodic departure time and the corresponding list of vehicles. Note that a vehicle list does not have to consist of only one vehicle, as is the case in this simple example, but could also consist of different vehicles.

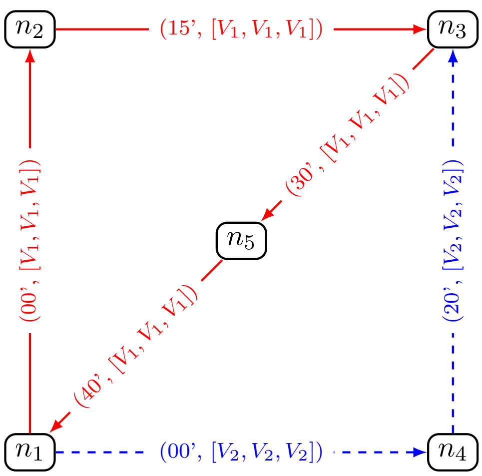


(a) Line network.

(b) Collapsed line network.

Figure 6: Line networks for Example 8.

The last figure, Figure 7, shows which edges of the collapsed line network can be joined to a new line. We get the old line l_3 as l_1^1 and all its subpaths as well as a new line l_2^1 with its subpaths in which the old lines l_1 and l_2 are contained.



Lines

$l_1^1 = (n_1, n_4, n_3)$

$l_1^2 = (n_1, n_4)$

$l_1^3 = (n_4, n_3)$

$l_2^1 = (n_1, n_2, n_3, n_5, n_1)$

$l_2^2 = (n_1, n_2, n_3, n_5)$

$l_2^3 = (n_2, n_3, n_5, n_1)$

$l_2^4 = (n_3, n_5, n_1)$

...

Figure 7: Coinciding labels.

The line pool generation is now complete and it remains to find a cost-minimal line concept based on this new line pool.

In the following theorem we show that Algorithm 1 finds a public transport plan that is consistent with the public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ used as input.

Theorem 9. *The public transport plan $(\mathcal{L}', \pi', \mathcal{V}')$ constructed by Algorithm 1 is consistent* *with the public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ used as input and line plan $\mathcal{L}'$ is feasible w.r.t the lower frequency bounds.*

Proof. The construction of the line network in lines 4 to 18 assigns an aperiodic departure time for each PTN edge $e \in P(r)$ covered by vehicle route $r \in \mathcal{V}$ that can be part of a trip according to the lower and upper bounds. In the collapsed line network constructed in line 20 to 33 these aperiodic coverings of edges are accumulated to a periodic one if the edge is covered in each period repetition at the same periodic time point. These collapsed edges are labeled by the list of vehicles which cover them in each period repetition. The construction of the paths in line 35 guarantees that each line is covered by one vehicle end-to-end in each planning period and that the corresponding timetable is feasible as transfers pose no restriction due to Section 2. Additionally, line concept $\mathcal{L}'$ is feasible as the minimal frequencies are respected due to line 38. It remains to show that the new vehicle schedule $\mathcal{V}'$ is feasible, that there exists a bijection $b\colon \mathcal{V} \to \mathcal{V}'$ of the vehicle routes and that the trips of $b(r)$ are part of the path $P(r)$ fitting to the duration of the connecting trips if applicable. As bijection b we map route r_i of vehicle V_i to the new route of vehicle V_i. Here, the new route of V_i consists of trips (p, l) where line l corresponds to a path in the collapsed line network with label VehList where vehicle V_i starts in period repetition p. This correspondence is unique as each edge $(e, \pi_i, \text{VehList})$ of the collapsed line network can only be part of one line, see line 39, and the covering of a PTN edge by Vehicle V_i in period repetition p, represented by line network edge $(e, \pi_i + p \cdot T, V_i)$, can only be part of one edge $(e, \pi_i, \text{VehList})$ of the collapsed line network, see line 32.

The construction of the collapsed line network also guarantees that all trips (p, l) in vehicle route $b(r)$ are part of $P(r)$ and that the corresponding aperiodic times coincide. The duration of trips that are part of an old connecting trip is fitting to the durations fixed in line 10 and therefore satisfies (8) and (9). The duration of connecting trips $((p_1, l_1), (p_2, l_2)) \in b(r)$, $r \in \mathcal{V}$ is feasible as well: Let v_1 be the last station of line l_1 and v_2 the first station in line l_2. Then there is a $v_1 - v_2$ path P_{v_1,v_2} which is part of $P(r)$. Covering P_{v_1,v_2} in vehicle route r takes at least as long as L_{l_1,l_2} which is defined as the length of the shortest $v_1 - v_2$ paths in the PTN according to the lower bounds on the drive times. Therefore, the trips (p_1, l_1) and (p_2, l_2) are compatible and the vehicle schedule $\mathcal{V}'$ is feasible as well. $\square$

To prove that this line concept is also cost-minimal under a technical assumption, we start by showing that the line pool constructed in Algorithm 1 contains all consistent lines.

Lemma 10. *Let the duration of the edges in connecting trips in $\mathcal{V}$ be uniquely determined by (8) and (9) and let for each edge $e \in E$ the aperiodic departure times $\tau_{(e,p,l)}$, $\tau_{(e,c)}$ be unique for all trips $(p, l) \in \mathcal{V}$ with $e \in (p, l)$ and connecting trips $c \in \mathcal{V}$ with $e \in c$, i.e., there is a most one departure using edge e at any point in time. Then all lines that are consistent with the public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ used as input are in the line pool $\mathcal{L}^0$ constructed in Algorithm 1.*

Proof. Note that due to the fixed duration of edges in connecting trips, the aperiodic departure times of edges in connecting trips can be uniquely determined. Due to the uniqueness of the departure times, the collapsed line network constructed in lines 20 to 33 is unique as well and thus especially the labels VehList.

Let l be a line that is not in $\mathcal{L}^0$, i.e., that is not constructed in line 36. We show that this line l is not consistent with $(\mathcal{L}, \pi, \mathcal{V})$.

At first we consider the case where each edge $e_i \in l$ corresponds to an edge $(e_i, \pi_i, \text{VehList}_i)$ in E_C. As $l \notin \mathcal{L}^0$ there either is no common label VehList for all edges $e_i \in l$ or the periodic departure times of two consecutive edges do not fit to the lower and upper bounds. As the aperiodic departure times of all edges are unique, the list of vehicles operating this edge in each planning period is unique and found by Algorithm 1. Therefore, differing labels for different edges show that line l is not covered by one vehicle end-to-end in each period repetition, i.e., the line is not consistent with $(\mathcal{L}, \pi, \mathcal{V})$. If the periodic departure times do not fit to the lower and upper bounds, the corresponding timetable π' is not feasible, i.e., line l is not consistent with $(\mathcal{L}, \pi, \mathcal{V})$.

We therefore only have to consider the case where at least one edge $e \in l$ has no corresponding edge in E_C. Due to the uniqueness of the aperiodic departure times, this means that for edge e there is no departure in each period repetition at the same periodic time. Thus, edge e cannot be part of a line consistent with public transport plan $(\mathcal{L}, \pi, \mathcal{V})$. □

Using Theorem 9 and Lemma 10, we show that the line plan constructed by Algorithm 1 is cost-minimal.

Theorem 11. *Let the duration of the edges in connecting trips in $\mathcal{V}$ be uniquely determined by (8) and (9) and let for each edge $e \in E$ the aperiodic departure times $\tau_{(e,p,l)}$, $\tau_{(e,c)}$ be unique for all trips $(p, l) \in \mathcal{V}$ with $e \in (p, l)$ and connecting trips $c \in \mathcal{V}$ with $e \in c$, i.e., there is a most one departure using edge e at any point in time. Then Algorithm 1 finds a public transport plan $(\mathcal{L}', \pi', \mathcal{V}')$ that is consistent with the public transport plan $(\mathcal{L}, \pi, \mathcal{V})$ used as input such that line* *plan $\mathcal{L}'$ is feasible w.r.t the lower frequency bounds and minimizes the line costs* (10)*.*

Proof. Due to Theorem 9, the public transport plan $(\mathcal{L}', \pi', \mathcal{V}')$ found by Algorithm 1 is consistent with $(\mathcal{L}, \pi, \mathcal{V})$ and line plan $\mathcal{L}'$ is feasible according to the lower frequency bounds. The line pool which is used for the optimization problem contains all consistent lines according to Lemma 10. Therefore, it only remains to show that the constraints of the optimization problem posed in lines 38 to 39 of Algorithm 1 are necessary.

The constraints posed in line 38 are necessary to ensure that $\mathcal{L}'$ is feasible w.r.t the lower frequency bounds. The constraints posed in line 39 are needed to ensure a bijection between the old and the new vehicle routes, i.e., they are necessary to guarantee a consistent line plan. Thus, the line plan constructed by Algorithms 1 is cost-optimal for all feasible line plans that can be extended to a consistent public transport plan. □

To show the optimality of the line plan constructed in Algorithm 1 we need two technical assumptions, namely that the duration of edges in connecting trips is unique and that for any edge there is at most one departure at any given point in time. The second assumption is easy to ensure by headway activities and is satisfied for realistic instances due to security concerns. On the other hand, the first assumption is unlikely to be satisfied for realistic instances as it allows for no buffer times in connecting trips. If it is not satisfied, the solution quality of Algorithm 1 depends on the durations fixed in line 10.

4 Iteration Scheme

As described in [Sch17], the re-optimization problems defined in Section 3 can be used in an iterative scheme to modify an existing public transport plan. In theory, the three algorithms `ReLinePlanning`, `ReTimetabling` and `ReVehicleScheduling` can be used in any order. However, not all concatenations of algorithms lead to improvements. In this section, we investigate the influence of different iteration schemes on both the passenger-oriented and the cost-oriented objective of the resulting public transport plan as described in Section 2.4. Remember that the passenger-oriented objective is to minimize the travel time of all passengers on shortest paths according to the timetable while the costs-oriented objective is to minimize the operational costs of the corresponding vehicle schedule.

At first, we consider the influence of the individual algorithms on the travel time and the operational costs. The influence of Algorithm `ReVehicleScheduling` can be determined most easily.

Lemma 12. *Let* $(\mathcal{L}, \pi, \mathcal{V})$ *be a public transport plan and* $(\mathcal{L}', \pi', \mathcal{V}')$ *the public transport plan after applying Algorithm* `ReVehicleScheduling` *to* $(\mathcal{L}, \pi, \mathcal{V})$*. Then the operational costs do not increase and the travel time is unchanged, i.e.,*

$$\text{cost}(\mathcal{V}') \leq \text{cost}(\mathcal{V})$$

$$\mathcal{R}_{\text{SP}}(\pi') = \mathcal{R}_{\text{SP}}(\pi).$$

Proof. Note that `ReVehicleScheduling` does not change the line plan or the timetable, i.e., $\mathcal{L}' = \mathcal{L}$ and $\pi' = \pi$. Therefore, we get $\mathcal{R}_{\text{SP}}(\pi') = \mathcal{R}_{\text{SP}}(\pi)$. Additionally, `ReVehicleScheduling` minimizes the operational costs and as $\mathcal{V}$ is a feasible solution of `ReVehicleScheduling` we get $\text{cost}(\mathcal{V}') \leq \text{cost}(\mathcal{V})$. □

Algorithm `ReTimetabling` has a clear effect on the travel time while its effect on the operational costs depends on their composition.

Lemma 13. *Let* $(\mathcal{L}, \pi, \mathcal{V})$ *be a public transport plan and* $(\mathcal{L}', \pi', \mathcal{V}')$ *the public transport plan after applying Algorithm* `ReTimetabling` *to* $(\mathcal{L}, \pi, \mathcal{V})$*. Then the travel time does not increase, i.e.,*

$$\mathcal{R}_{\text{SP}}(\pi') \leq \mathcal{R}_{\text{SP}}(\pi).$$

If the duration based costs are neglected, i.e., for $\text{cost}_{\text{time}} = 0$*, the operational costs are not changed, i.e.,*

$$\text{cost}(\mathcal{V}') = \text{cost}(\mathcal{V}).$$

Proof. Note that Algorithm `ReTimetabling` does not change the line plan, i.e., $\mathcal{L}' = \mathcal{L}$ and the composition of the vehicle routes in $\mathcal{V}'$ is the same as in $\mathcal{V}$. However, the start and end times of trips and connecting trips may change.
$\mathcal{R}_{\text{SP}}(\pi)$ evaluates the travel time of all passengers on shortest path w.r.t timetable π and Algorithm `ReTimetabling` sets the passenger weights w according to the same paths. As Algorithm `ReTimetabling` optimizes the travel time of the passengers on these fixed paths, i.e.,

$\mathcal{R}_{\text{fix}}(\pi', w)$, and π is a feasible solution, we get

$$\mathcal{R}_{\text{SP}}(\pi) \geq \mathcal{R}_{\text{fix}}(\pi', w).$$

By rerouting the passenger on optimal routes according to timetable π' we get

$$\mathcal{R}_{\text{SP}}(\pi) \geq \mathcal{R}_{\text{fix}}(\pi', w) \geq \mathcal{R}_{\text{SP}}(\pi').$$

When evaluating the costs of a public transport plan without regarding the duration-based costs and without depots, we get

$$\text{cost}(\mathcal{V}) = \sum_{r\in\mathcal{V}} \text{cost}_{\text{len}} \cdot \Big(\sum_{\substack{\text{trip}\\ t\in r}} \text{len}_t + \sum_{\substack{\text{connecting trip}\\ c\in r}} \text{len}_c \Big) + \text{cost}_{\text{veh}} \cdot |\mathcal{V}|.$$

As the composition of the vehicle routes in $\mathcal{V}$ and $\mathcal{V}'$ are the same, i.e., they contain the same trips and the same connecting trips, we get

$$\begin{aligned}
\text{cost}(\mathcal{V}) &= \sum_{r\in\mathcal{V}} \text{cost}_{\text{len}} \cdot \Big(\sum_{\substack{\text{trip}\\ t\in r}} \text{len}_t + \sum_{\substack{\text{connecting trip}\\ c\in r}} \text{len}_c \Big) + \text{cost}_{\text{veh}} \cdot |\mathcal{V}| \\
&= \sum_{r\in\mathcal{V}'} \text{cost}_{\text{len}} \cdot \Big(\sum_{\substack{\text{trip}\\ t\in r}} \text{len}_t + \sum_{\substack{\text{connecting trip}\\ c\in r}} \text{len}_c \Big) + \text{cost}_{\text{veh}} \cdot |\mathcal{V}'| \\
&= \text{cost}(\mathcal{V}').
\end{aligned}$$

□

Example 14 shows that for positive duration based costs, i.e., for $\text{cost}_{\text{time}} > 0$, the operational costs can be increased by Algorithm `ReTimetabling`.

Example 14. Consider an event-activity network as given in Figure 8. Suppose there are W passengers transferring at station n_1 from line l_2 to line l_1 and W passengers transferring from line l_1 to line l_2 at station n_2. Suppose that in the original timetable the departure of line l_2 at station n_2 is schedule shortly before the arrival of line l_1 at the same station such that the transfer takes almost a full planning period. Then by delaying the departure of line l_2 at station n_2, the transfer time gets shorter improving the travel time of the passengers but the duration of line l_2 increases, leading to higher operational costs.

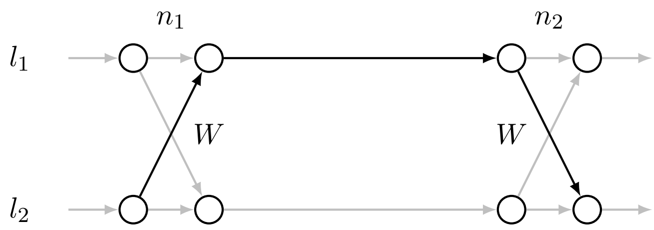
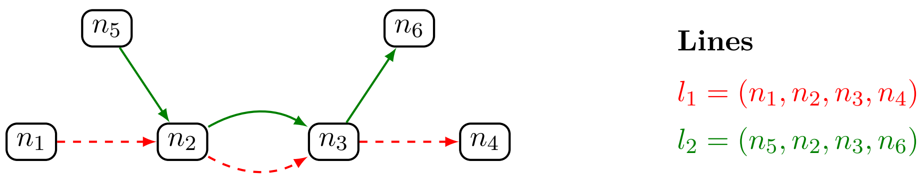


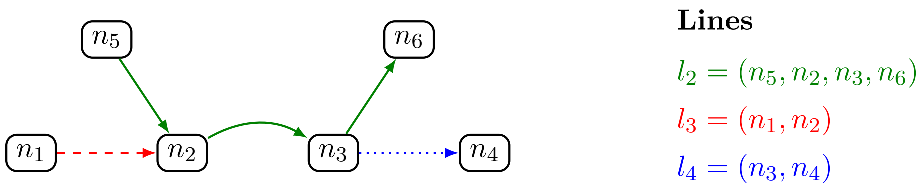
Figure 8: Excerpt of the event-activity network.

The effects of Algorithm `ReLinePlanning` are the most difficult to determine. First note that the travel time can be increased as shown in Example 15.

Example 15. Consider the PTN and line plan given in Figure 9a. After applying Algorithm `ReLinePlanning` we can get the situation depicted in Figure 9b, if the minimal frequency of edge (n_2, n_3) is 1 and the fixed costs of a line are relatively low.



(a) Line plan before applying Algorithm `ReLinePlanning`.



(b) Line plan after applying Algorithm `ReLinePlanning`.

Figure 9: Line plans for Example 15.

This means that passengers driving from n_1 to n_4 have to transfer at station n_2 and station n_3 and therefore might have significantly higher travel times.

It remains to examine the influence of Algorithm `ReLinePlanning` on the operational costs.

Lemma 16. *Let $(\mathcal{L}, \pi, \mathcal{V})$ be a public transport plan and $(\mathcal{L}', \pi', \mathcal{V}')$ the public transport plan after applying Algorithm* `ReLinePlanning` *to $(\mathcal{L}, \pi, \mathcal{V})$. Then the operational costs do not increase, i.e.,*

$$\text{cost}(\mathcal{V}') \leq \text{cost}(\mathcal{V}).$$

Proof. We analyze the operational costs of $(\mathcal{L}', \pi', \mathcal{V}')$ by looking at the different parts of the operational costs separately. We write

$$\text{cost}(\mathcal{V}) = \text{cost}_{\text{time}} \cdot \sum_{r \in \mathcal{V}} \texttt{duration}(r) + \text{cost}_{\text{len}} \cdot \sum_{r \in \mathcal{V}} \text{len}(r) + \text{cost}_{\text{veh}} \cdot |\mathcal{V}|$$

where $\texttt{duration}(r)$ describes the duration of vehicle route r and $\text{len}(r)$ its length.

From Definition 6 we get bijection b of the vehicle routes. Thus we get

$$|\mathcal{V}| = |\mathcal{V}'|. \tag{11}$$

The duration of a vehicle route $r = ((p_1, l_1), \dots, (p_n, l_n))$, is defined by the duration of its trips and connecting trips, i.e.,

$$\texttt{duration}(r) = \sum_{i=1}^{n} (\texttt{end}_{p_i, l_i} - \texttt{start}_{p_i, l_i}) + \sum_{i=1}^{n-1} (\texttt{start}_{p_{i+1}, l_{i+1}} - \texttt{end}_{p_i, l_i}) = \texttt{end}_{p_n, l_n} - \texttt{start}_{p_1, l_1}$$

Route r and route $b(r)$ differ from one another as not all edges in r have to be covered by $b(r)$. Especially, the route might start later or end earlier. Thus we get

$$\texttt{duration}(r) \geq \texttt{duration}(b(r)). \tag{12}$$

The length of a vehicle route $r = ((p_1, l_1), \dots, (p_n, l_n))$, is defined by the length of its trips and connecting trips.

$$\text{len}(r) = \sum_{i=1}^{n} \text{len}_{l_i} + \sum_{i=1}^{n-1} D_{l_i, l_{i+1}}$$

With $D_{l_i, l_{i+1}}$ being the length of a shortest path and the definition of $P(r)$ in the beginning of Section 3.3 we get

$$\text{len}(r) = \Big(\sum_{i=1}^{n} \text{len}_{l_i} + \sum_{i=1}^{n-1} D_{l_i, l_{i+1}} \Big) = \sum_{e \in P(r)} \text{len}_e.$$

From Definition 6 we get that the paths of all trips of $b(r)$ are contained in the path $P(r)$ but connecting trips of $b(r)$ use a shortest path. With the triangle inequality we get

$$\text{len}(r) \geq \text{len}(b(r)). \tag{13}$$

Combining equations (11), (12) and (13) we get

$$\begin{aligned}
\operatorname{cost}(\mathcal{V}) &= \sum_{r\in\mathcal{V}} \texttt{duration}(r) + \sum_{r\in\mathcal{V}} \operatorname{len}(r) + \operatorname{cost}_{\text{veh}} \cdot |\mathcal{V}| \\
&\leq \sum_{r\in\mathcal{V}} \texttt{duration}(b(r)) + \sum_{r\in\mathcal{V}} \operatorname{len}(b(r)) + \operatorname{cost}_{\text{veh}} \cdot |\mathcal{V}'| \\
&= \operatorname{cost}(\mathcal{V}').
\end{aligned}$$

□

We now use Lemmas 12, 13 and 16 to formulate convergence results for iteratively applying the Algorithms `ReLinePlanning`, `ReTimetabling` and `ReVehicleScheduling`. As the travel time is more difficult to improve, we can only guarantee convergence for applying `ReTimetabling` and `ReVehicleScheduling` although the objectives of both algorithms differ.

Theorem 17. *Let P_0 be a feasible public transport plan with travel time t_0. Let P_i, $i \in \mathbb{N}^+$, be a public transport plan derived from P_{i-1} by applying either* `ReTimetabling` *or* `ReVehicleScheduling` *and let t_i be the travel time of P_i. Then the sequence of travel time values $(t_i)_{i\in\mathbb{N}}$ decreases monotonically and converges.*

Proof. As all feasible activity durations are positive, the sequence is bounded from below by 0. From Lemmas 12 and 13 we get that the travel time is not increased by `ReTimetabling` while `ReVehicleScheduling` has no influence on it. Therefore, $(t_i)_{i\in\mathbb{N}}$ is monotonic and bounded and converges by the monotone convergence theorem, see e.g. [Sut09]. □

For the operational costs, we can guarantee convergence if duration based costs are neglected, i.e., if $\operatorname{cost}_{\text{time}} = 0$.

Theorem 18. *Let P_0 be a feasible public transport plan with operational costs c_0 where duration based costs are neglected, i.e., with* $\operatorname{cost}_{\text{time}} = 0$. *Let P_i, $i \in \mathbb{N}^+$, be a public transport plan derived from P_{i-1} by applying either* `ReLinePlanning`, `ReTimetabling` *or* `ReVehicleScheduling` *and let c_i be the operational costs of P_i. Then the sequence of operational cost values $(c_i)_{i\in\mathbb{N}}$ decreases monotonically and converges.*

Proof. As all vehicle schedules have positive costs, the sequence is bounded from below by 0. From Lemmas 12, 13 and 16 we get that the operational costs are not increased by `ReLinePlanning` and `ReVehicleScheduling` as well as `ReTimetabling` if duration based costs are neglected, i.e., if $\text{cost}_{\text{time}} = 0$ is satisfied. Therefore, $(c_i)_{i\in\mathbb{N}}$ is monotonic and bounded and converges by the monotone convergence theorem, see e.g. [Sut09]. $\square$

Especially, we get convergence for travel time and costs if duration based costs are neglected, i.e., if $\text{cost}_{\text{time}} = 0$ is satisfied, and only `ReTimetabling` and `ReVehicleScheduling` are applied.

Corollary 19. *Let P_0 be a feasible public transport plan with travel time t_0 and operational costs c_0 where duration based costs are neglected, i.e., $\text{cost}_{\text{time}} = 0$ is satisfied. Let P_i, $i \in \mathbb{N}^+$, be a public transport plan derived from T_{i-1} by applying either* ***`ReTimetabling`*** *or* ***`ReVehicleScheduling`****. Let t_i and c_i be the travel time and the operational costs of P_i, respectively. Then both the sequence of travel time values $(t_i)_{i\in\mathbb{N}}$ and the sequence of operational cost values $(c_i)_{i\in\mathbb{N}}$ decrease monotonically and converge.*

Proof. The sequence $(t_i)_{i\in\mathbb{N}}$ converges by Theorem 17 and $(c_i)_{i\in\mathbb{N}}$ converges by Theorem 18. $\square$

5 Computational Experiments

We test the iterative scheme to modify an existing public transport plan on two different data sets. The first one, `grid`, is a benchmark instance described in [FHSS17], while the second one, `regional`, is a close-to real-world data set derived from the regional train system in Lower Saxony, Germany. The public transportation network of `grid` is a 5×5 grid network consisting of 25 stations and 40 edges. The PTN of `regional` consists of 35 stations and 36 edges. Both networks are depicted in Figure 10.

(a) PTN of data set `grid`.

(b) PTN of data set `regional`.

Figure 10: PTNs of data sets `grid` and `regional`.

We use data set `grid` as a case study with a fixed OD matrix described in [FHSS17]. For data set `regional` we apply the algorithms to ten different demand scenarios and report the average increases and decreases of the objectives.

The computations are conducted on a compute server with an Intel(R) Xeon(R) X5675 CPU @ 3.07 GHz and 132 GB of RAM.

To test the iterative algorithms, we at first compute an initial public transport plan using the LinTim software framework, see [SAP+18]. Here, the cost model of line planning, see [CvDZ98, Sch12], the standard periodic timetabling problem, see [SU89], and a cost-oriented vehicle scheduling model without a depot, see [BK09], are used. The timetabling problem is solved by a modulo simplex heuristic, see [GS13]. Afterwards, we apply one of the following iteration schemes:

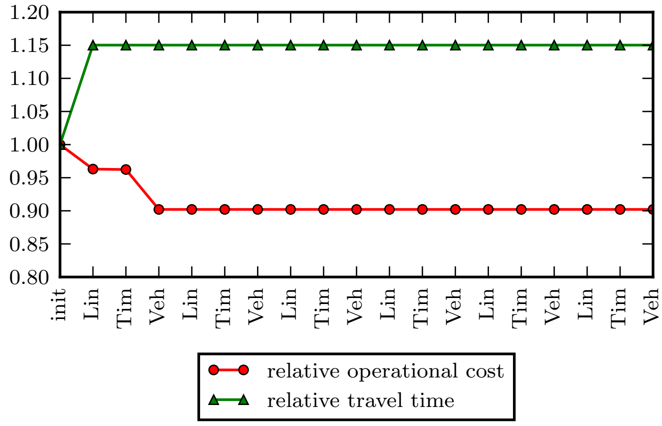
`forward` Iteratively compute a public transport plan by applying the Algorithms `ReLinePlanning`, `ReTimetabling` and `ReVehicleScheduling`.

`backward` Iteratively compute a public transport plan by applying the Algorithms `ReVehicleScheduling`, `ReTimetabling` and `ReLinePlanning`.

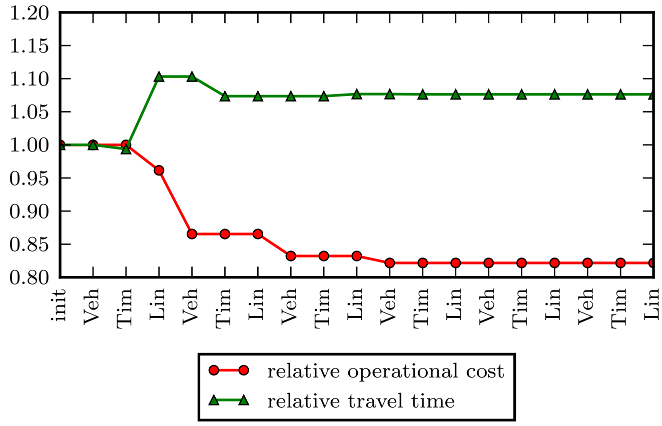
`mixed` Iteratively compute a public transport plan by applying the Algorithms `ReLinePlanning`, `ReTimetabling`, `ReVehicleScheduling` and again `ReTimetabling`.

`passenger convenience` Iteratively compute a public transport plan by alternately applying the Algorithms `ReTimetabling` and `ReVehicleScheduling`.

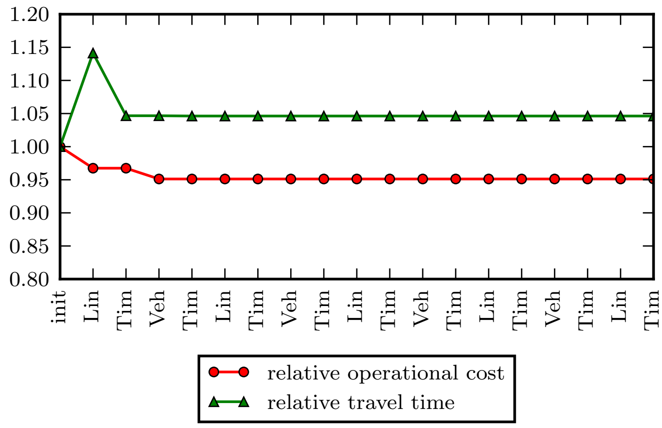
We use two different cost parameter sets for the computations, either `normal` which reflects a close-to real-world cost evaluation or `convergence` which differs from `normal` by setting the duration based costs to 0, i.e., setting $\text{cost}_{\text{time}} = 0$. Note that due to Theorem 18, cost parameter set `convergence` guarantees the convergences of the operational costs. For each public transport plan we compute the travel time on shortest paths according to the corresponding timetable and the operational costs depending on the cost parameter set that was used for the computation. Instead of the absolute values, we plot the relative values depending on the travel time and operational costs of the initial public transport plan, respectively. For both data sets, the runtime of each iteration is in the range of minutes. However using larger data sets for long-distance networks increases the runtime dramatically as not only the network size but also the trip length increases which both contribute to the problem size. Note that for Algorithm `ReTimetabling` we use the current timetable as starting solution to speed up the computation.



(a) Iteration scheme `forward`.



(b) Iteration scheme `backward`.



(c) Iteration scheme `mixed`.

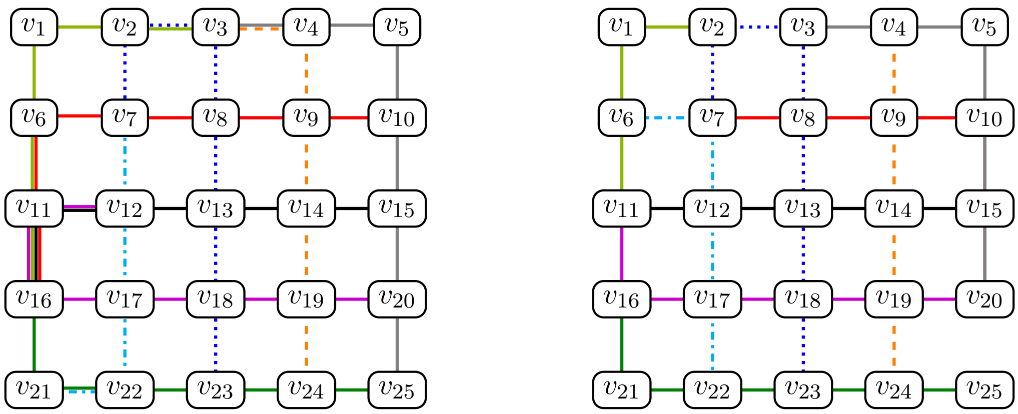
Figure 11: Applying different iteration schemes for data set `grid` with cost parameter set `normal`.

For data set `grid` we compare the influence of the different iteration schemes for cost parameter set `normal` on the convergence and the solution quality.

Figure 11 shows that although convergence is not guaranteed, both travel time and operational costs do not change anymore after a few iterations. However, the travel time does not decrease monotonically. Especially for iteration scheme `backward`, depicted in Figure 11b, the travel time increases multiple times. Note that although for the operational costs monotonicity and convergence is not guaranteed as duration based costs are not neglected, i.e., for $\text{cost}_{\text{time}} > 0$, the costs decrease monotonically for all iteration schemes considered here.

The solutions found by the different iteration schemes vary in respect to travel time and operational costs. While `backward` yields the highest operational cost decrease of 18%, the travel time increases by 8%. On the other hand `mixed` yields a lower decrease of 5% of the initial operational costs but the increase in travel time is much lower, with only 5%. Depending on the preference corresponding to the trade-off between travel time and operational costs, both solutions are interesting options. In contrast, the solution for iteration scheme `forward` is clearly worse than the one for iteration scheme `backward`, as both the decrease in operational costs is lower with 10% and the increase in travel time is higher with 15%.

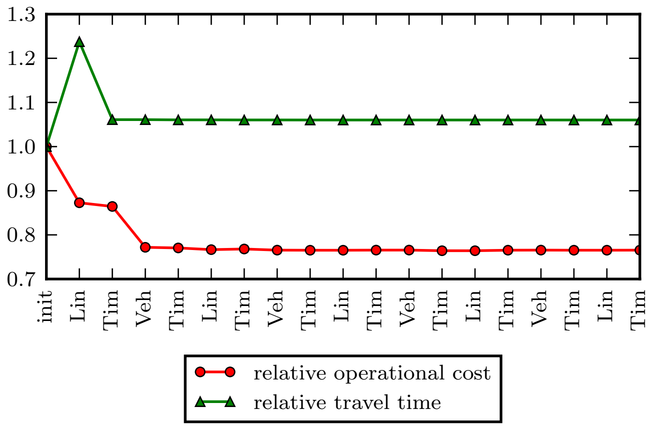
Figure 12 shows the impact of convergence scheme `backward` on the line plan. The coverage of the PTN edges decreases, yielding the large improvements in operational costs but also the increase in travel time. While often lines are simply shortened, see, e.g. the orange dashed line or stay the same, see, e.g. the dark blue dotted line, also new lines are formed. The cyan dash-dotted line now directly connects station v_6 to the stations v_{12}, v_{17} and v_{22}. In the initial line plan there is at least one transfer necessary to connect these stations.



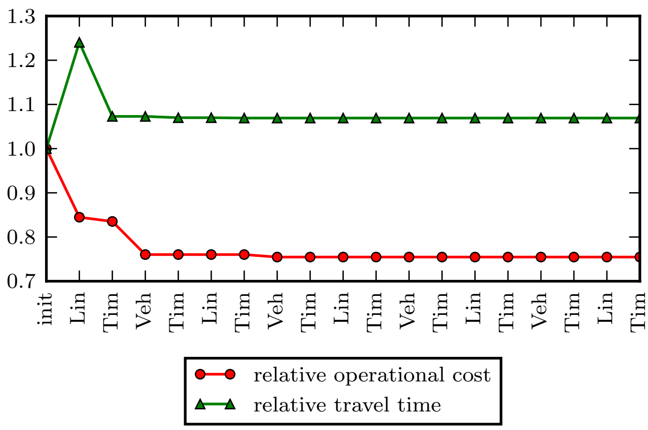
(a) Initial solution. (b) After applying iteration scheme `backward`.

Figure 12: Line concepts of data set `grid`.

For data set `regional`, we get even better results when considering iteration scheme `mixed` for the cost parameter sets `normal` and `convergence`. Although monotonically decreasing costs are only guaranteed for cost parameter set `convergence`, Figure 13 shows that the costs



(a) Cost parameter set `normal`.

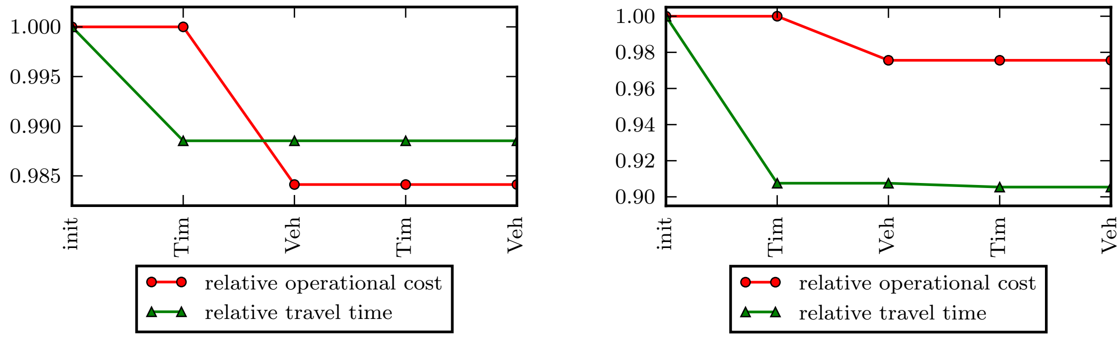


(b) Cost parameter set `convergence`

Figure 13: Applying iteration scheme `mixed` for data set `regional` with different cost parameter sets.

decrease monotonically for both parameter sets. This can also be observed for data set `grid`, see Figure 11, showing that in practice Algorithm `ReTimetabling` does not often increase the costs even if duration based costs are considered. Furthermore, the costs decrease is even higher than for data set `grid` with 24% decrease for parameter set `normal` and 25% for parameter set `convergence`. Even though for both parameter sets the travel time does not decrease, the increase is relatively low compared to the reduction in operational costs with 6% and 7% for cost parameters sets `normal` and `convergence`, respectively. For parameter set `normal` there even is one instance where the travel time is slightly reduced by 2% while the operational costs are also reduced by 25%.

When considering iteration scheme `passenger convenience` with cost parameter set `convergence`, as depicted in Figure 14, we see that both the travel time and the operational costs decrease monotonically as expected due to Corollary 19. Note that here only the first two iterations are illustrated as no further changes occur in the later iterations. For data set `grid` the improvement is relatively small with 1% decrease of travel time and 2% decrease in operational costs. However, for data set `regional` the travel time is decreased significantly by 9% with a small improvement of the operational costs by 2%. This makes the solution clearly preferable to the initial solution and makes for an interesting additional choice to the solution found by iteration scheme `mixed` for `regional` with the same cost parameter set `convergence` with lower costs but significantly higher travel time.



(a) Data set `grid`.

(b) Data set `regional`.

Figure 14: Applying iteration scheme `passenger convenience` with cost parameter set `convergence`.

In order to investigate the influence of the initial solution on the quality of the solution found the iteration schemes, we apply the iteration schemes `forward`, `backward` and `mixed` to two different initial solutions for data set `grid` with cost parameter set `normal`. *Initialization cost* is the initial solution described above, computed by using the cost model of line planning, a periodic timetabling model and a standard vehicle scheduling model. *Initialization direct* uses the direct travelers model of line planning, see [Bus98], combined with the same timetabling and vehicle scheduling models. Figure 15 shows that the solutions derived from applying the iteration schemes to initialization cost and initialization direct differ. Especially, the set of solutions found for initialization direct is preferable to the set of solutions found for initialization cost as for each solution derived from initialization cost there exists a strictly dominating solution derived from initialization direct. However, the solution found by the iterative schemes are all similar in travel time and operational costs, with average travel times varying from 23 to 25.8 and average operational costs varying from 890 to 984, although the initial solutions differ a lot with average travel times of 22.29 and 18.65 and average operational costs of 1144 and 2051.28, respectively.

Figure 15 especially shows that the iteration schemes `forward`, `backward` and `mixed` are mainly focused on minimizing operational costs instead of minimizing travel time.

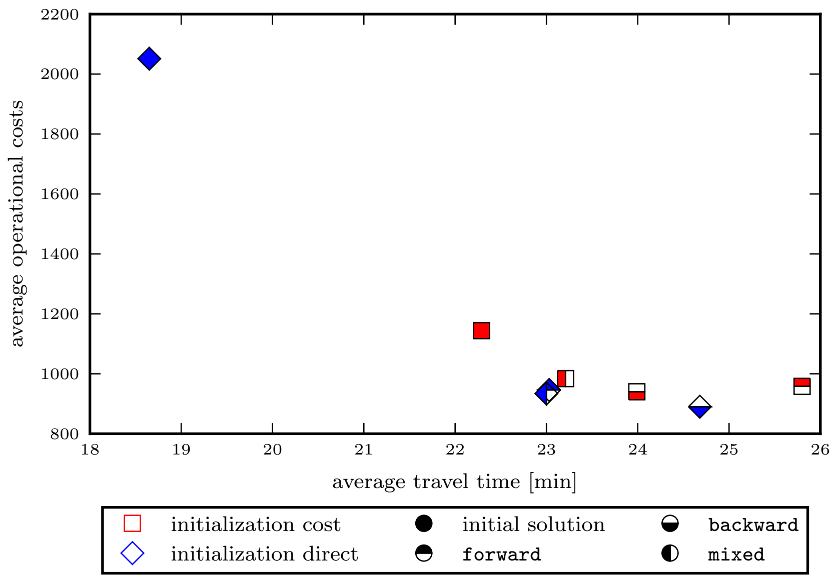


Figure 15: Comparing different initial solutions for iteration schemes `forward`, `backward` and `mixed` on data set `grid` with cost parameter set `normal`.

6 Outlook

There are several possible extensions to the models presented in this paper. First of all, the experiments show a clear tendency towards optimizing the cost, due to both vehicle scheduling and line planning both using costs as an objective. But especially for line planning, multiple possible models and objective functions are described in the literature. These could be adapted to serve as the last step of Algorithm 1, replacing the cost-optimization. This may lead to more balanced solutions, favouring the quality for the passengers.

Another possibility is to embed the iterative scheme in the eigenmodel approach discussed in [Sch17]. The problems described here form the "inner circle" of this model, see Figure 16. Therefore, it would be interesting to model the remaining problems that are not researched yet to create an meta-model for public transport planning. Several paths in the eigenmodel, representing different sequential solution approaches, are already researched (e.g. [MS09, PSSS17]), but there are still several challenges to discuss, considering new and already researched solution

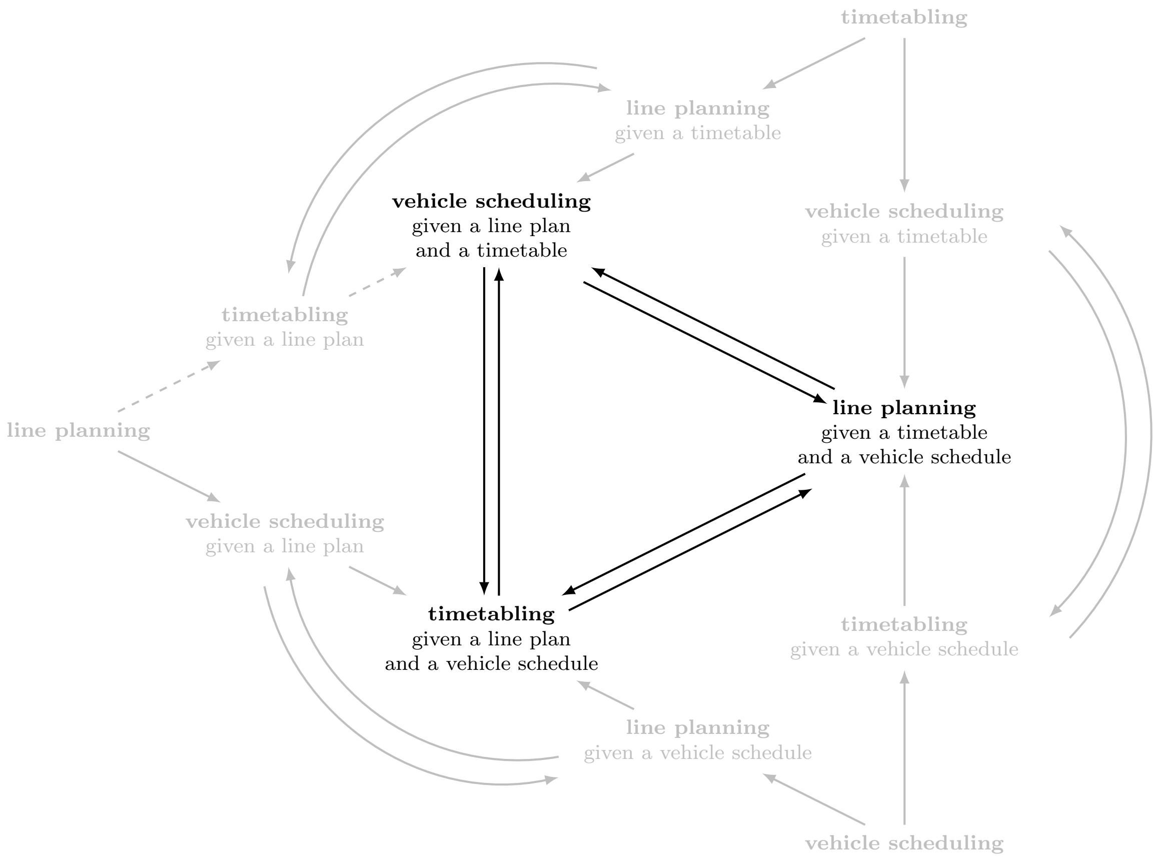


Figure 16: Algorithmic scheme called eigenmodel. Nodes represent algorithms while edges represent possible concatenations of them. All possible sequential approaches to finding a public transport plan are shown, where the algorithms presented above are depicted in black. The classical sequential approach to public transport planning is depicted with dashed edges. For more information, see [Sch17].

approaches. In the end it would be interesting to determine good paths in the eigenmodel which approximate an integrated approach to public transport planning. One possibility would be to use machine learning techniques in developing a meta-algorithm for the planning process.

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