PARAMETER IDENTIFICATION FOR THE HELMHOLTZ EQUATION FROM LIMITED OBSERVATIONS IN HELIOSEISMOLOGY

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Abstract

The goal of local helioseismology [1] is to recover 3D quantities in the solar interior (density, sound speed, flows, ...) from observations of the line-of-sight velocity $\psi(\mathbf{r},t)$ at surface points $\mathbf{r}$ and observation time t. The basic input data are the cross-correlations between velocities at any two points of the solar surface $C(\mathbf{r}_1,\mathbf{r}_2,t)$ which can be written in the Fourier domain as $C(\mathbf{r}_1,\mathbf{r}_2,\omega) = \psi(\mathbf{r}_1,\omega)^* \psi(\mathbf{r}_2,\omega)$. We assume for simplicity that ψ satisfies the scalar wave equation

$$L_{c,\rho} := -\frac{\omega^2+2i\omega\gamma}{\rho c^2}\psi - \nabla.\left(\frac{1}{\rho}\nabla\psi\right) = s \tag{1}$$

where ρ is the density, c the sound speed, γ represents wave attenuation and s is a stochastic source of excitation. Supposing that the source power is equipartitioned, then the expectation of the cross-covariance $E[C(\mathbf{r}_1,\mathbf{r},\omega)]$ is proportional to the imaginary part of the Green's function $\mathrm{Im}[G(\mathbf{r}_1,\mathbf{r},\omega)]$ [2]. Note that this situation is encountered in various fields of seismology or in ocean tomography (see e.g. the review [3]).

We want to solve the following type of inverse problems *(IP)*: Recover the parameters α (c or ρ), knowing the observations Im[G] at the solar surface where G is the Green's function associated to the operator $L_{c,\rho}$ with boundary conditions.

Theoretically, it is known that a parameter α is uniquely determined from the measurements at one frequency of the full Green's function at the surface and that the two parameters c and ρ can be recovered from the measurements at two different frequencies [4]. But we are not aware of any theoretical results when only the imaginary part of the Green's function is observed.

Here, we investigate numerically this situation by solving the inverse problem *(IP)* using the Iteratively Regularized Gauss Newton Method (IRGNM) with Conjugate Gradient (CG) inner iterations. The forward problem (1) with boundary condition is solved using the Finite Element Method. The gradient and its adjoint that are also required to solve *(IP)* with iterative methods can both be obtained by solving an equation similar to (1) where only the right hand side is modified resulting in a fast implementation. First, we focus on the case where the density is constant in order to study how much information is lost by looking only at the imaginary part instead of the full Green's function. Then, we discuss the difficulties to solve this problem in the solar context where the density is varying of several orders of magnitude close to the surface increasing the level of ill-posedness of the problem.

References

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