Class Announcement
Winter Semester 2012/13

Compressed Sensing

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The key observation that forms the basis for the theory of compressed sensing is that many natural signals are approximately *sparse* in suitable bases, that is, they can be well-approximated by linear combinations of just a few of the basis elements. The aim of this theory is to use this fact already for reducing the amount of measurements to capture a signal rather than in a post-processing step after the full signal has been acquired. More precisely, a suitable, small set of linear measurements can allow to recover a sparse signal via convex optimization algorithms such as ℓ_1-minimization.

The focus of this class will be on which measurements are suitable to guarantee for successful recovery. In particular, it will turn out that matrices which have the *Restricted Isometry Property* (RIP) will play a crucial role, that is, $A \in \mathbb{C}^{m\times N}$, $m \ll N$, such that any submatrix consisting of s of its columns is well-conditioned. The smallest embedding dimension m that allows for this property is of order $m \log\left(\frac{N}{s}\right)$, but no deterministic constructions are known to find RIP matrices are known. Rather the smallest known embedding dimensions are achieved by random matrices. We will show that matrices of this embedding dimension whose rows are independent subgaussian vectors have the RIP. When the rows of the matrix are a random subset of the rows of the discrete Fourier matrix, the embedding dimension can be chosen of an order which is optimal up to logarithmic factors.

The proofs of these results will involve techniques from probability theory in Banach spaces such as chaining and Dudley entropy integrals, which will be introduced in the course of this class.

Requirements: Measure theory, linear algebra, probability theory

References:
[1] H. Rauhut. *Compressive sensing and structured random matrices,* in *Theoretical Foundations and Numerical Methods for Sparse Recovery*, ed. M. Fornasier, volume 9 of Radon Series Comp. Appl. Math., pages 1–92
http://rauhut.ins.uni-bonn.de/LinzRauhut.pdf

[2] R. Vershynin. *Introduction to the non-asymptotic analysis of random matrices*, in *Compressed Sensing, Theory and Applications*, ed. Y. Eldar and G. Kutyniok. Cambridge University Press, 2012, pp. 210–268
http://www-personal.umich.edu/∼romanv/papers/non-asymptotic-rmt-plain.pdf

[3] F. Krahmer, S. Mendelson, and H. Rauhut. *Suprema of chaos processes and the restricted isometry property.* Preprint, 2012.

Time: Monday, 10.15–12.00 **Room:** HS 6