Class Announcement

Summer Semester 2012

Non-Asymptotic Random Matrix Theory

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A random matrix $A \in \mathbb{R}^{m \times N}$ is a matrix-valued random variable. For example, the entries may be i.i.d. scalar Gaussian or Bernoulli random variables, but random matrices with dependent entries will also be considered. There are many classical results about the asymptotic behaviour of the spectrum of such matrices, like for example Wigner's semicircle law. However, it has recently become important to also understand the *non-asymptotic* spectral behaviour of random matrices. A typical question of interest is for example the following: Consider random matrices A of size $m \times N$, how much do the singular values of their realizations differ from the predicted asymptotic behaviour?

The class will, in large parts, follow the lecture notes by Roman Vershynin [1].

We will start with non-asymptotic deviation estimates for random variables in one dimension. We introduce subgaussian and subexponential random variables and random vectors as well as the isotropic random vectors. The concepts then appear in the study of random matrices, mainly matrices with independent rows or columns. We will prove results on the tail behaviour of their maximal singular values for different types of distributions for the row/column vectors. Furthermore various applications will be discussed, including dimension reduction [2] and compressed sensing.

Requirements: Measure theory, linear algebra, probability theory

References:

[1] Vershynin, R.: *Introduction to the non-asymptotic analysis of random matrices*, in *Compressed Sensing, Theory and Applications*, ed. Y. Eldar and G. Kutyniok. Cambridge University Press, 2012, pp. 210–268
http://www-personal.umich.edu/∼romanv/papers/non-asymptotic-rmt-plain.pdf

[2] Krahmer, F. and Ward, R.: *New and improved Johnson-Lindenstrauss embeddings via the Restricted Isometry Property*, SIAM J. Math. Anal. 43(3), 2011, 1269–1281. http://na.math.uni-goettingen.de/pdf/KW11.pdf

Time: Monday, 10.15–12.00 **Room:** MN 55