Numerical reconstructions for the Calderón problem in 3D

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In the technology for medical imaging called Electrical Impedance Tomography the goal is to recover the electric properties in a body from electrostatic measurements on the surface of the body. Mathematically the basic problem is known as the Calderón problem (or the so-called inverse conductivity problem), and it was formulated by Calderón in 1980. The problem concerns the unique determination and reconstruction of an electric conductivity distribution (a coefficient in a PDE) in a bounded domain from knowledge of the Dirichlet-to-Neumann (or Voltage-to-Current) map on the boundary of the domain. The inverse problem is known to be a notoriously difficult and severely ill-posed inverse problem.

During the last three decades the Calderón problem has been studied intensively. Of particular interest for applications is the reconstruction aspect of the three dimensional inverse problem. In theory such a reconstruction algorithm was given in the mid 1980s by Sylvester-Uhlmann, Nachman and Novikov. The algorithm is based on ideas from inverse scattering and attacks the full non-linear problem at once. Only recently the development of a numerical and practical reconstruction algorithm following the same principles was initiated.

In this talk I will describe the theory behind the algorithm and discuss the numerical implementation. In addition I will illustrate the performance of the method on several numerical examples and demonstrate how regularization of the ill-posed problem is naturally build into the algorithm.