Introduction to Mathematica

In those two sessions we are going to explore Mathematica

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* Based on
M100 : An introduction to Mathematica
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1 First Five minutes with Mathematica

- The ***shift + return*** operation sends instructions from the interface where you are typing to the engine of Mathematica for processing
- You will notice right away two peculiarities of the syntax
  - The **names of all Mathematica functions and constants** begin with **capital letters**. Mathematica is completely case –sensitive

    Example

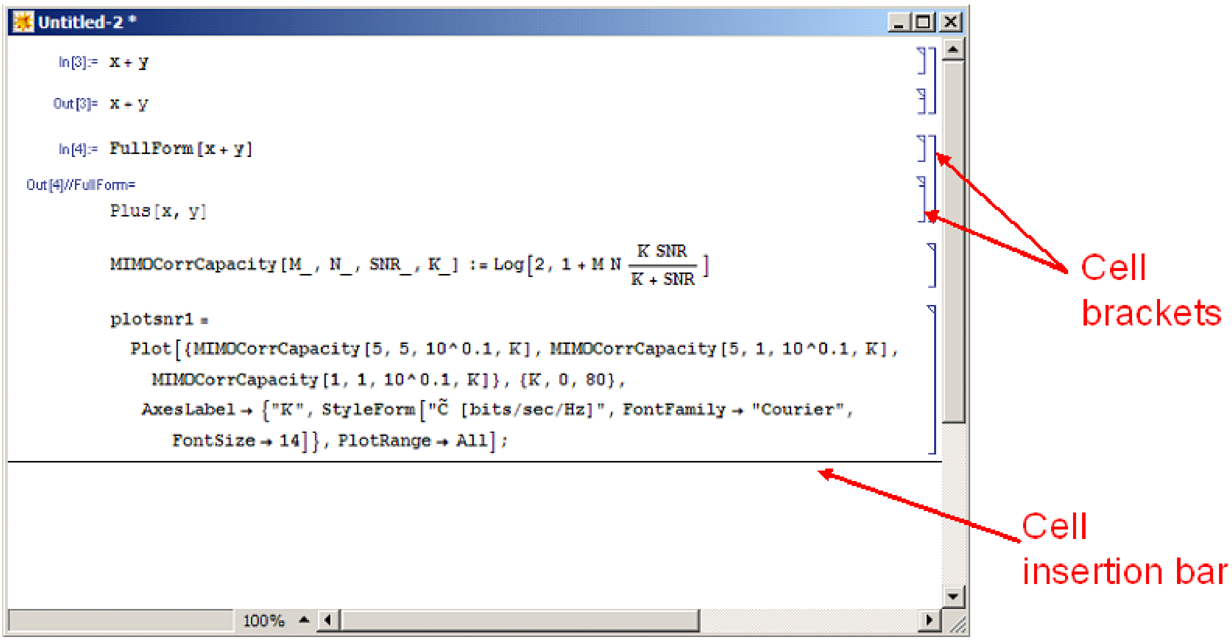
    ```
    sin[pi / 3]
      vs
    Sin[Pi / 3]
    ```

  - **Brackets** types

    [ ] – are always used to enclose the arguments of functions
    { } – are used in Mathematica to group pieces of data together – forming structures called lists
    ( ) – are used only for grouping expressions

2 Our First Encounter with Mathematica

- Our work is written in a **notebook**, this document is a notebook, just like working with Word or other word processing environment
- The basic element of a notebook is a **cell**, where each cell is enclosed with the blue bracket on the right
- We make our calculation in the notebook, then we send the calculation to the **kernel**. The kernel perform the calculation and sends the result back and present it within a new cell



- Each input cell is numbered sequantially
- Use **;** when you want the output not to be printed on the screen, yet an output cell is generate

  Example

  2 + 1

  2 + 2;

- The percent sign, *%*, is used to mean “the last output”

Example

  %

  % + 1

```
%%
```

Exercise

1 Type in and test all the code in section 2

3 Arithmetic

At its simplest, Mathematica can be thought of as a highly sophisticated calculator. Like a calculator, it does arithmetic

Example

```
2 + 5
```

```
2 * 5
```

```
2^5
```

```
100 !
```

```
Sin[Pi / 3]
```

```
Sqrt[50]
```

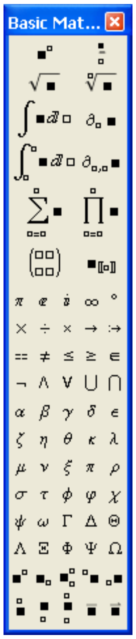
```
2^(1 + 4)
```

```
Log[2, %]
```

■ A space can implicitly mean multiplication

3.1 Basic Math Input – Pallet

Some of the above calculations can be laid out in a way that corresponds more closely to conventional mathematical notation by using the Basic Input palette



■ Find it in the Palettes menu
■ This is especially useful when you need to build up large expressions

Sin[30 °] + Cos[60 °]

2×5

2^5

2^{1+4}

$\text{Sin}\left[\frac{\pi}{3}\right]$

$\text{Sin}\left[\frac{i\pi}{3}\right]$

$\sqrt{50}$

■ Numerical values of these constants are given for any required accuracy using the **N** command

$N\left[\sqrt{50}, 1\right]$

$N\left[\sqrt{50}, 5\right]$

```
N[π, 50]
```

■ known constants

```
Pi

E

E // N

π

ⅇ

ⅈ

I

∞
```

Exercise

1 Type in and test all the code in section 3
2 Try the following :

```
(3 - 2 I) * (1 + I)

(1 + 5 I)^2

Conjugate[2 - 5 I]

Abs[12 - 5 I]

Arg[1 + √3 I]

Exp[1 + I]
```

4 Algebra & Calculus

As well as being an arithmetical calculator, Mathematica is also an algebraic one
Example

```
Expand[(x + 2 y)^2 (x - 3 y)^5]

Factor[%]
```

■ Equations in Mathematica are set up using a double equals sign '=='
■ The **Solve** command tries to find exact solutions to algebraic equations
Example

```
Solve[x^2 - 3 x + 2 == 0, x]
```

```
Solve[x^4 + 3 x^3 + 5 x^2 - 11 x + 2 == 0, x]

Solve[{x + 4 y == 5, 2 x - y == 8}, {x, y}]
```

■ **Mathematica will perform calculus operations too**

Example

```
D[x^2, x]
```

$\partial_x x^2$

Example

```
D[y x^2, x]
```

Example

```
Integrate[x ^3, x]
```

$\int x^3 \, dx$

Example

```
Integrate[x^3, {x, -3, 3}]
```

$\int_{-3}^{3} x^3 \, dx$

Example

$\int \int y^2 \, x \, dx \, dy$

■ **The NIntegrate command uses numerical integration methods : essential for those cases where analytical approaches would be difficult or inappropriate**

Example

```
NIntegrate[Sin[x], {x, 0, 1}]
```

■ **some functions can be numerically or symbolically**

Example

```
Integrate[Sin[x], {x, 1, 2}]

NIntegrate[Sin[x], {x, 1, 2}]
```

Exercise

1 Type in and test all the code in section 4

2 Use Mathematica to express $\frac{1}{I + 1}$ in terms of its real and imaginary parts

3 Try the following

Apart$\left[\frac{2\,x}{\left(1+x^2\right)\,(1+x)}\right]$

Together[%]

Expand$\left[(3+2\,x)^2\,(x+2\,y)^2\right]$

Collect[%, x]

Expand$\left[(3+2\,x)^2\,(x+2\,y)^2\right]$

Simplify[%]

Cancel$\left[\frac{x\text{^}2+5\,x+6}{x+3}\right]$

Numerator$\left[\frac{x\text{^}2+5\,x+6}{x+3}\right]$

4 open the Algebraic Manipulation palette (under Palettes in the File menu). This palettehas the setting "Evaluate in Place". To find out what this means, first type, without evaluating, $\int \frac{2+3\,x+x^2}{2+2\,x+x^2}\,dx$, Then select the fraction inside the integral, and click on the Apart[■] button. With the same piece of text selected, click on Together[■]. Try using Expand[■] , and so on

Investigate the use of the Evaluate in Place instruction, under Evaluation in the menu

5 Type

Sum[1 / r^2, {r, 1, 6}]

or

$$\sum_{r=1}^{6} \frac{1}{r^2}$$

Try summing from 1 to 20

Try summing from 1 to n,

and from 1 to infinity (Infinity in Mathematica, or use the ∞ symbol from the Basic Input palette)

6 solve

- the ordinary differential equation, $\frac{d^2\,y}{dx^2}+y=0$, using the DSolve function
- this differential esubject to the inital conditions {y (0) = 1, y ' (0) = 1}

5 Assignment

name expressions which you will want to use again

Example

```
expression1 = 2 x / ((1 + x^2) (1 + x))

expression2 = Apart[expression1]

expression3 = Together[expression2]

TrueQ[expression1 == expression3]
```

■ Notice the final Q in the function name : this is a convention for logical functions (those whose output is True or False)
■ We can make expression1 into an unassigned symbol again by clearing its value
Example

```
Clear[expression1]
```

■ Quiting our Mathematica session (**Evaluation → Quit_Kernel**) will clear all assignments pretty effectively, and leave everything clear for our next go

■ it is sometimes more appropriate to avoid **global** assignments of this type and opt for **local** substitution instead
Example
Compare the following pieces of code,
each of which aims at finding the value of the expression $x^2 - 5x + 9$ at $x = 3$
Here is the first one

```
Clear[x]

x = 3

x^2 - 5 x + 9

Clear[x]
```

Here is the second

```
x^2 - 5 x + 9 /. x → 3
```

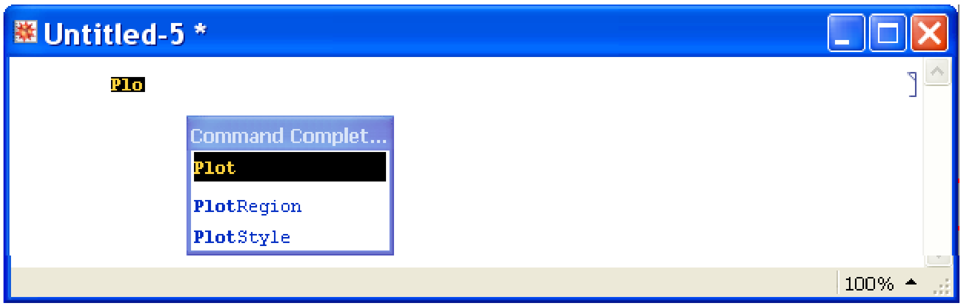
6 Common Mistakes

■ names of all Mathematica functions and constants begin with capital letters
■ () instead of []
■ [] instead of ()
■ NIntegrate[Sin[x], x, 0, 1] → NIntegrate[Sin[x], {x, 0, 1}]
■ Plot[xSin[x], {x, 0, 1}] → Plot[x_Sin[x], {x, 0, 1}]
■ These are not the same → f (x), f[x]
■ Solve[x^2 - x = 1, x] → Solve[x^2 - x == 1, x]

7 Functions

7.1 Built in functions

- Mathematica has a big library of built in functions
- a built in functions is allways capitalized
- 'Ctrl + K' completes the function name



- Getting help
  - Use the special query character '?Sqrt'
  - You can do “wildcard” '? Plot * ' or '? * Plot'
  - Emphasize function name and press F1

Exercise

1 search for the function Plot in the help
2 Type '?Plot', '?Plot * ', '? * Plot' and plot a sinus

7.2 Defining your own functions

Working with your own functions in Mathematica always involves two distinct stages

- first you define the function, using the **underscore** character and **:=**

Example

```
Clear[x, f]
```

f[x_] := $x^2 - 5x + 9$

- after Mathematica has lerant this new function, and for the rest of your session, you can use it in just the same way as inbuilt functions

Example

```
f[3]
```

```
f[z]
```

```
D[f[z], z]
```

- use **Module** to create a nested function

```
ff[x_] := Module[{}, {a = x, b = x^2}]
```

```
ff[3]
```

■ use ' **Initialization Cell** ' to upload varaibles from memory after leaving Kernel (create new cell → Cell → Cell_Properties → InitializationCell)

Exercise

1 Type in and test all the code in section 7.2

2 define your own function. this function recives two varaibles (x , y) and return thier mutiplication and deviations values. Use InitializationCell with the vales {a = 2, b = 3}.

8 Procedural Programming

■ Inequalities

```
x == y, x > y, x < y, x ≤ y, x ≥ y
```

■ Logical connectives

- ■ Logical AND expr1 && expr2 && ... AND[expr1, expr2, ...]
- ■ Logical OR expr1 || expr2 || ... OR[expr1, expr2, ...]

■ Conditions

If, Switch

Example

```
f[x_] := If[x > π, Print[x, " is larger than π"], Print[x, " is not larger than π"]]
```

try

```
f[E]
f[8]
```

■ Loops & flow controll

While For Return

9 Graphics

```
Plot[Sin[x], {x, -2 π, 2 π}]

sinPlot = Plot[Sin[x], {x, -2 π, 2 π}]

Plot[Tooltip[{Sin[x], Cos[x]}], {x, 0, 10}]

Plot[Tan[x], {x, 0, 20}, Exclusions → {Cos[x] == 0}]

Manipulate[Plot[Sin[c * x], {x, 0, 10}], {c, 1, 5}]

Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}]
```

Exercise

1 Type in and test all the code in section 9