

## Geometric Analysis of Vision Cues for the Flare

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**Abstract.** For executing the “flare” or “roundout” right before landing an aircraft, there are various practical suggestions in the literature, based on visual perception. These are analyzed here by looking at their basic geometric principles. Then their differences, advantages, and disadvantages can be discussed on a solid common ground, allowing pilots to decide which technique they may prefer.

### 1 Introduction

What is a “flare”?

*When the airplane, in a normal descent, approaches within what appears to be 10 to 20 feet above the ground, the round out or flare is started. This is a continuous process until the airplane touches down on the ground [6].*

*In the flare, the nose of the plane is raised, slowing the descent rate and therefore, creating a softer touchdown, and the proper attitude is set for touchdown [10].*

The web has plenty of suggestions for the “right” way to decide to flare, because the flare altitude is not easy to assess if there is no ground radar or a callout. One needs visual cues for estimating the altitude, and these vary considerably:

- *When you see the runway “zoom” in your windscreen, it’s time to flare [2].*
- *... starting the roundout when the trapezoidal shape of the runway seems to suddenly get wider [3].*
- *... you will see the aircraft sinking at some point, and the runway rising. This is when you flare the aircraft [9].*

- *The flare is initiated when, on a stable approach, the pre-determined impact point, appearing to move downward from the aim point, reaches the cockpit cut-off angle and disappears from view under the aircraft [7].*

These rules-of-thumb differ considerably, but what is behind them?

Here, some simple mathematics is applied to the problem of flaring, hopefully leading to a better understanding of various practical rules. Several visual cues are analyzed, their advantages and disadvantages are discussed. Whatever technique is preferred, it boils down to training the estimation of vision angles or distances in some way or other. The usual training programs for pilots seem to do exactly that, without mentioning.

## 2 Flare Parameters

The standard assumption will be to fly on the usual glide slope of  $3^\circ$ , roughly equivalent to a 1:20 descent ratio. The glide slope ends at the “slope aiming point”. To avoid misunderstandings, this is not necessarily the “Aiming Marker” [5] on the runway, and not the “view aiming point” that will occur later when dealing with vision, i.e. the point the pilot tries to focus on when deciding to flare. For a standard ILS approach, the slope aiming point will usually be the Aiming Marker on the runway, while in General Aviation the slope aiming point may be the runway threshold.

On the standard glide slope, the altitude  $h$  is closely connected by  $h = x \cdot \tan 3^\circ$  or  $x \approx 20 \cdot h$  to the distance  $x$  to the slope aiming point. This means that the decision to flare at a certain point can be determined via altitude or distance to the slope aiming point. In general, altitude will be more difficult to estimate than distance, unless there is a ground radar or a callout. In addition, misestimating altitude by 10 feet is as influential as misestimating distance by 200 feet. This makes distance estimation preferable.

Missing the flare will roughly result in a crash after about 3 seconds. A large aircraft at 130 knots IAS and a typical flare altitude [1] of 30 ft crashes after 2.73 seconds, while a General Aviation aircraft at 60 knots IAS and a flare altitude of 15 ft crashes after 2.96 seconds. Therefore the time scale for the flare is in the range of about three seconds.

The flare or roundout should be a continuous process [6], and it amounts to reducing the flight path angle from  $3^\circ$  to zero. If this is done one degree per second, it takes three seconds to arrive at horizontal flight. See Figure 1 for the two typical

cases described in the previous paragraph. The left case flares at 30 ft altitude at 130 knots and has the Aiming Markers 1000 ft on the runway as slope aiming points, while the right case flares at 15 ft at 60 knots, aiming at the runway threshold. Horizontal flight is theoretically reached at 13 or 7 ft, respectively, but the “wheel crossing height” [4] must be accounted for.

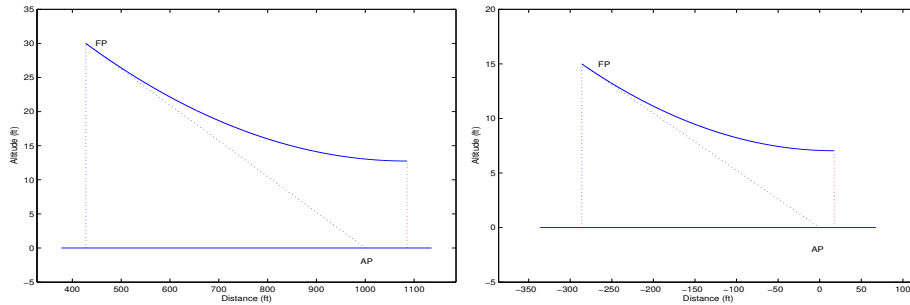


Figure 1: Roundout for large (left) and small aircraft (right)

The essence of these arguments is that one needs some way of either estimating the flare altitude or the distance to the aiming point. This has to be done within very tight limits for time, altitude, and distance. Therefore, some additional known quantitative information is needed. It comes easier in distances, like runway lengths or distances of markers on the runway, and hardly as direct altitude information, as long as there is no ground radar available. Therefore visual cues for the flare should focus on points with known distances.

### 3 Longitudinal Visual Estimation

Here, we ignore looking left or right, just into the direction of the glide path down to the aiming point. The counterpart, Transversal Visual Estimation, will follow in Section 4.

In nearly all situations, at least two points at known distance in longitudinal direction can be spotted on a runway: the threshold and the runway end, plus some additional markers on larger runways. The pilot can realize the angle under which these markers visually appear. But by some background mathematics suppressed here, this information is still not enough. Therefore this view angle should be connected to the axis of the pilot’s view on the glide slope, as in Figure 2.

A simple way to estimate an altitude on a glideslope of angle  $\beta$  by a visual cue from the pilot’s seat is to look down at a well defined angle  $\alpha$  to a well-defined

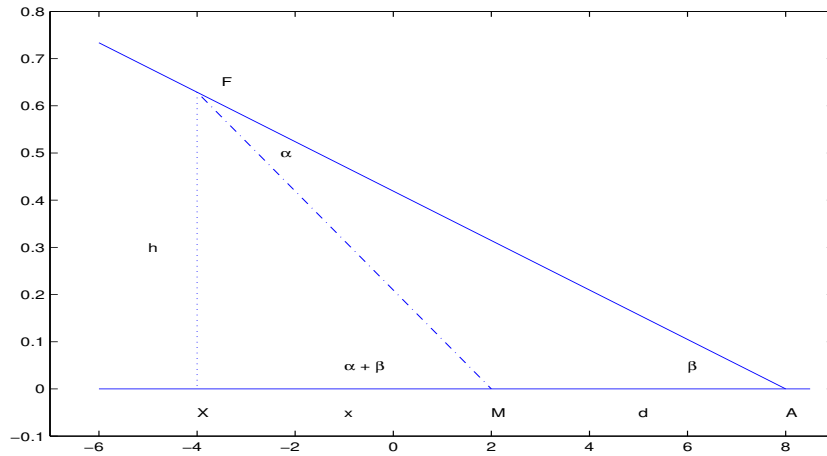


Figure 2: Flaring at  $F$  when seeing  $M$  at angle  $\alpha$ . This is the Jacobson flare.

point  $M$  at distance  $d$  from the slope aiming point  $A$ . The simplest practical idea is to choose the angle  $\alpha$  in such a way that any point below the angle  $\alpha$  gets invisible if the pilot's seat position, the windshield geometry and the angle of attack on the glideslope are fixed. Then the flare is initiated exactly when "flying over  $M$ ", in the sense that  $M$  gets invisible from the pilot's seat while flying down the glideslope with a fixed seat position and exactly towards the slope aiming point  $A$ . Whatever  $\alpha$  and  $d$  are, the flare point gets reproducible. This is the basic idea of the "Jacobson flare" [8], but there are variations to be added later.

If  $x$  is the distance of the flare ground position  $X$  to  $M$ , and the altitude at  $X$  on the glideslope is  $h$ , Figure 2 shows that  $h$  is determined uniquely. To see this by geometry, start at the slope aiming point  $A$  and draw the glide slope at angle  $\beta$ . Then draw the point  $M$  at distance  $D$  on the runway. Finally, go up at an angle  $\alpha + \beta$  from  $M$  to intersect the glideslope at the point  $F$ . From  $F$ , the point  $M$  appears at the angle  $\alpha$  below the vision line along the glideslope.

Analytically, the equations

$$\tan(\alpha + \beta) = \frac{h}{x}, \quad \tan(\beta) = \frac{h}{x+d}$$

allow to solve for  $h$  via some calculations. But we may simplify this a bit by replacing the tangent values by descent rates

$$\frac{1}{a} := \tan(\alpha + \beta) = \frac{h}{x}, \quad \frac{1}{b} := \tan(\beta) = \frac{h}{x+d}$$

to finally get

$$h = \frac{d}{b-a}$$

more easily, from  $x = ah$ ,  $bh = x + d = ah + d$ . The standard glideslope has  $\tan(\beta) \approx 1/20$  and if we assume  $\tan(\alpha + \beta) = 1/10$ , we get  $b = 20$  and  $a = 10$ , then  $h = d/10$ .

Assume that the flight operations manual prescribes a flare altitude  $h$  for a certain aircraft. Then the aircraft- and pilot-dependent  $\alpha$  determines  $d$  for the fixed glideslope angle via  $d = (b-a)h$ . Therefore the pilot needs a marker at a very specific distance  $d$  before the slope aiming point  $A$ . This distance has to be estimated somehow, maybe via stripes on the runway, if there are any at known positions.

Arguing the other way round, the pilot may choose known markers on the runway, e.g. the threshold and the Aiming Markers at distance 1000 ft, and then the pilot must remember the angle  $\alpha$  under which these markers should be visible at flare time, maybe by the distance their view positions span on the windscreen.

In the first case, the pilot has to estimate a distance on the runway, in the second a view angle or, equivalently, a distance on the windscreen. It is not clear which case is easier to handle.

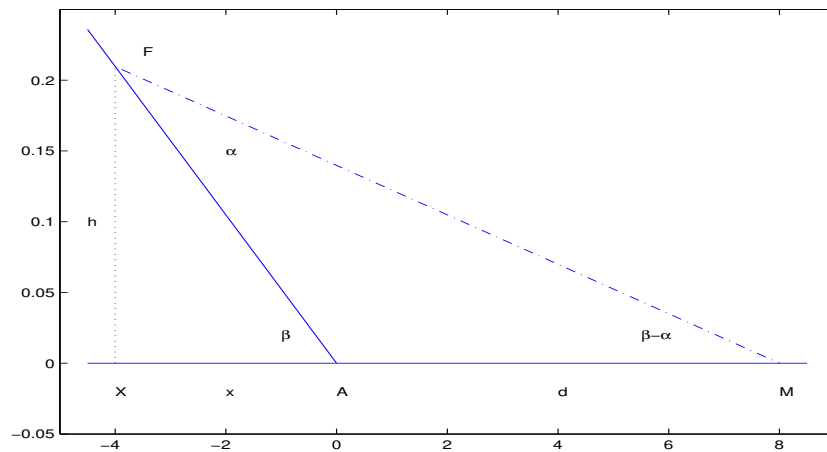


Figure 3: Flaring at  $F$  when fixing the view to  $M$ , while the glideslope aims at  $A$ .

A variation occurs if only the threshold and the end of the runway can serve as visual cues. The aiming point  $A$  of the glideslope should then be the threshold, but the view of the pilot should focus at the end  $M$  of the runway. See Figure 3. The

mathematical analysis is very similar, but not exactly the same, as readers might find out. In practice, this variation does not allow the “fly over a certain point” strategy. The pilot must estimate an angle or a point distance on the windscreen. But if airport restrictions allow, the pilot can aim the glide slope at some point  $A$  at a fixed known distance before the threshold, and then “flare when flying over  $A$ ” if  $A$  is placed properly. When training repeatedly at a fixed runway, this can work fine, but it is not of much help when arriving at unfamiliar runways.

A third variation, included in Jacobson’s original publication [7], does not refer to viewing at all. It is a way to let  $d$  be aircraft-dependent. The idea is to specify a height  $z$  that is roughly the height of the pilot’s eye over the wheelbase, and to draw a line parallel to the glideslope at distance  $z$  (the “wheel path”) to meet the runway at  $M$ . See Figure 4. On the standard glideslope, this means  $d = 20z$  and forces the pilot to estimate a specific distance of points on the runway some way or other. The flare altitude then is  $h = 20z/(b - a)$  and cares for the aircraft size. For prescribed flare altitudes in Flight Crew Operations Manuals, there might be a conflict.

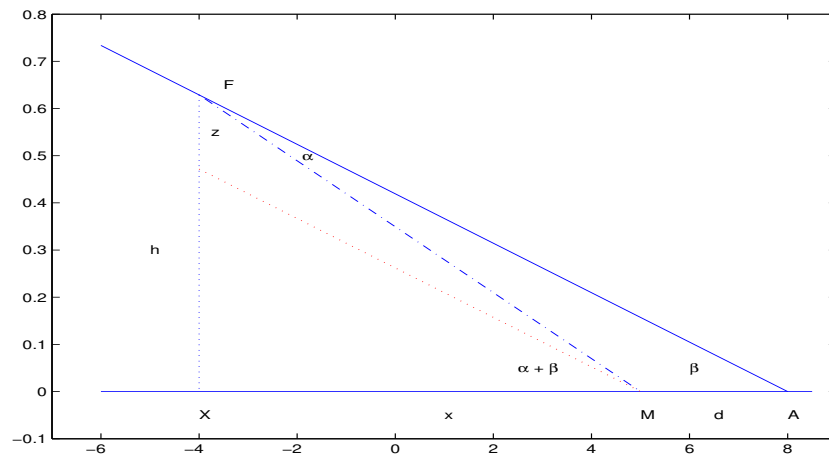


Figure 4: Determining the point  $M$  and the distance  $d$  via  $z$ .

Summarizing, all of this depends heavily on practically useful and reliable choices for the distance  $d$  and the angle  $\alpha$ . Furthermore, the pilot’s seat position and view direction must be carefully fixed to get reliable results. The “fly over  $M$ ” technique of Figure 2 takes a good choice of  $\alpha$ , but is then forced to estimate distances between two points on the runway, where usually only one point is visually given. If one works with two given points on the runway, as in Figure 3, one has to estimate an angle from the view direction, which will not be easy to do in general.

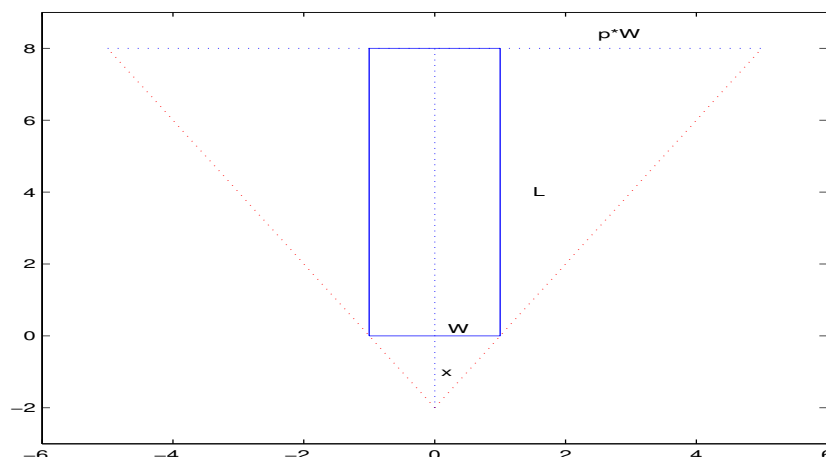


Figure 5: When viewed from distance  $x$  to the first line, the first line appears to be  $p$  times longer than the second.

It is unclear whether angles or distances are easier to estimate, and there will be strong personal preferences or dislikes.

## 4 Transversal Visual Estimation

This will now consider the effect that a runway visually widens dramatically when coming to the flare point. The information now is not only longitudinal, i.e. in flight direction, but also transversal. Since glide angles are small, we work in the ground plane and ignore the  $3^\circ$  angle between the ground plane and the horizontal plane containing the glide slope. The committed relative error will be less than 1%, because  $\cos(3^\circ) = 0.9986$ .

Assume the pilot looks at two parallel lines at distance  $L$  on the runway, e.g. one being the threshold and the other being either the runway end or the line between the Aiming Markers 1000 ft behind the threshold. Let the runway width be  $2W$ . If  $x$  is the distance of the viewer to the first line, the first line will appear to be longer, say  $p$  times longer than the second. See Figure 5 for illustration. Then

$$\frac{W}{x} = \frac{p \cdot W}{x + L}$$

allows to ignore  $W$ , leading to the distance

$$x = \frac{L}{p - 1}.$$

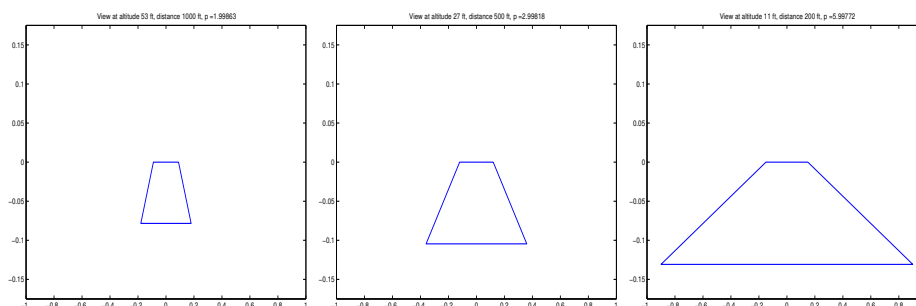


Figure 6: A view at  $3^\circ$  glide angle onto the first 1000 ft of the runway, at distances 1000, 500, and 200 feet.

Assume that the second line is at the Aiming Markers at the standard distance of 1000 ft from the threshold. If the pilot sees the threshold 6 times wider than the line between the Aiming Markers, the distance to the threshold is  $x = 1000/5 = 200$  ft. If the threshold appears only 3 times wider, the distance is 500 ft. See Figure 6 for illustration. This is the exact view from the glide angle of  $3^\circ$ , and therefore the multipliers  $p$  are not 2, 3, and 6, but 1.99863, 2.99818 and 5.99772, accounting for tilting the ground or the glideslope plane by  $3^\circ$ . Note that the view direction aims consistently at the distant line which should be between the Aiming Markers, while the near line is the runway threshold. The plot axes are not to scale, but this has no influence on the relations of the lengths. However, the axis scales are kept the same for the three cases.

Figure 7 deals with a typical General Aviation runway of 2500 feet viewed at distances 1250, 500, and 100 feet from the threshold. Again, the view direction is focused on the far line, this time the end of the runway. Because the far line is very short for long runways, estimation of the factor  $p$  may be difficult. All runways should have a marker line at 1000 ft from the threshold to make this distance estimation technique trainable and runway-independent.

This technique does not need angle estimation, and it is largely independent of changes in seat position and view direction. However, it requires estimating the **relative** sizes of **two** transversal point distances.

Theoretically, one can get away with just one transversal angle estimation combined with a known runway width. In Figure 5, the view angle  $2\alpha$  for the first line is given by

$$\tan \alpha = \frac{W}{x}$$



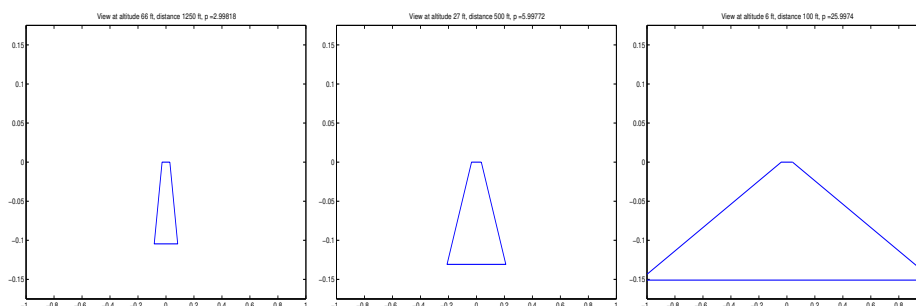


Figure 7: A view at  $3^\circ$  glide angle onto a  $2500 \times 75$  ft runway, at distances 1250, 500, and 100 feet. Factors  $p$  are about 3, 6, and 26, with very small discrepancies due to the glideslope.

such that a consistent flare can be executed as soon as a certain line across the runway spans a certain view angle. If the seat position is fixed, such an angle can be correlated to a horizontal distance on the windscreen. This strategy will depend on the runway width  $W$ , while  $W$  cancels out when two parallel lines are used and their relative lengths are estimated.

## 5 Conclusions

There are various mathematically supported strategies for using visual cues that may help to find the correct flare point in a reliable and repeatable way. They have advantages and disadvantages, and pilots will individually prefer one over another. The purpose of this article is to offer the full choice, with explanations and comparisons based on geometry.

It can be assumed that any pilot training will implicitly train the perception and estimation of distances and angles, like any good training in tennis or basketball. There might be an independent interest in testing and training vision issues of this kind, e.g. for screening tests. And, the stated geometric principles may help to design assistant landing systems based on fixed cameras. Besides simulators, there might be separate sessions for “visual training” that help to interpret visual cues correctly.

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