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Impedance tomography on trees

collaboration with the project-group for non-destructive diagnose of trees of the Fachhochschule Hildesheim/Holzminden/Göttingen.

Aim of the project:
To
find the areas of decay and/or damage to the tree to evaluate the tree and his danger to the surrounding and so decide if the tree must be chop down.

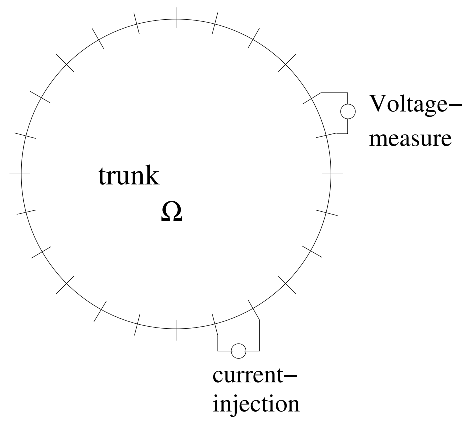

Figure 1: configuration

Problem: Consider reconstruction of the conductivities in the domain on the basis of the measured impedances on the outer boundary.

Measurement-device and test-situation

Ring with 24 electrodes which slightly penetrate the bark to ensure good coupling and to avoid the insulating bark.

Figure 2: measurement-device with test situation

Mathematical model

Consider the time-independent Maxwell equations especially Ampère's law and Faraday's law (induction law) and using that the current is divergence free and that a potential u exists.

further assumptions:

- punctual injection and measurement of current and voltage.
- There exist no current flow through the boundary.

with this assumptions we come to an unique neumann boundary-value problem of the following form:

(direct problem)

Find potential u in Ω with

$$\begin{aligned}\nabla \cdot (\sigma \nabla u) &= I\delta(\cdot - x_s) - I\delta(\cdot - x_q),\\ \sigma \frac{\partial u}{\partial \nu} &= 0,\\ \int_\Omega u \,\mathrm{d}\, x &= 0.\end{aligned}$$

(inverse problem)

with given $u|_{\partial\Omega}$, I, x_s and x_q find σ.

where x_s is the location of the drain and x_q is the location of the source of the current.

FEM-direct solver

Assume the boundary $\partial\Omega$ to be of piecewise C^2 with well-behaved edges and corners. Change to weak formulation of the problem. let $u \in H^1(\Omega)$ and $v \in H^1(\Omega)$.

the problem in the weak formulation becomes:

$$\int_\Omega \nabla v \cdot \sigma \nabla u \,\mathrm{d}\, x = Iv(x_s) - Iv(x_q)$$

because of H^1 we must approximate the δ-function:

$$\int_\Omega \nabla v \cdot \sigma \nabla u \,\mathrm{d}\, x = \int_\Omega I(x) v \,\mathrm{d}\, x \tag{1}$$

numerical Implementation in 2D:

With the choice of triangles as finite elements and a basis of piecewise linear functions ($\psi_k : k = 1, \ldots, N$), u and v can be represented as

$$u = \sum_{k=1}^{N} \alpha_k \psi_k \quad v = \sum_{j=1}^{N} \beta_j \psi_j$$

with N being the number of elements. with this representation and chosing $\beta_j = \delta_{ij}$ eq. (1) becomes:

$$\sum_{k=1}^{N} \int_\Omega \alpha_k \nabla \psi_i(x) \cdot (\sigma \nabla \psi_k(x)) \,\mathrm{d}\, x = \int_\Omega I(x) \cdot \psi_i(x) \,\mathrm{d}\, x$$

This can be written as

$$\sum_{k=1}^{N} (\alpha_k)_k \cdot (S(\psi_i, \psi_k))_{i,k} = \int_\Omega I(x) \cdot \psi_i(x) \, \mathrm{d}\, x \;\; (i = 1, \dots, N)$$

with

$$(S(\psi_i, \psi_k))_{i,k} = \int_\Omega \nabla \psi_i(x) \cdot (\sigma \nabla \psi_k(x)) \, \mathrm{d}\, x$$

being the so-called *stiffnessmatrix*.

Example for the direct solver

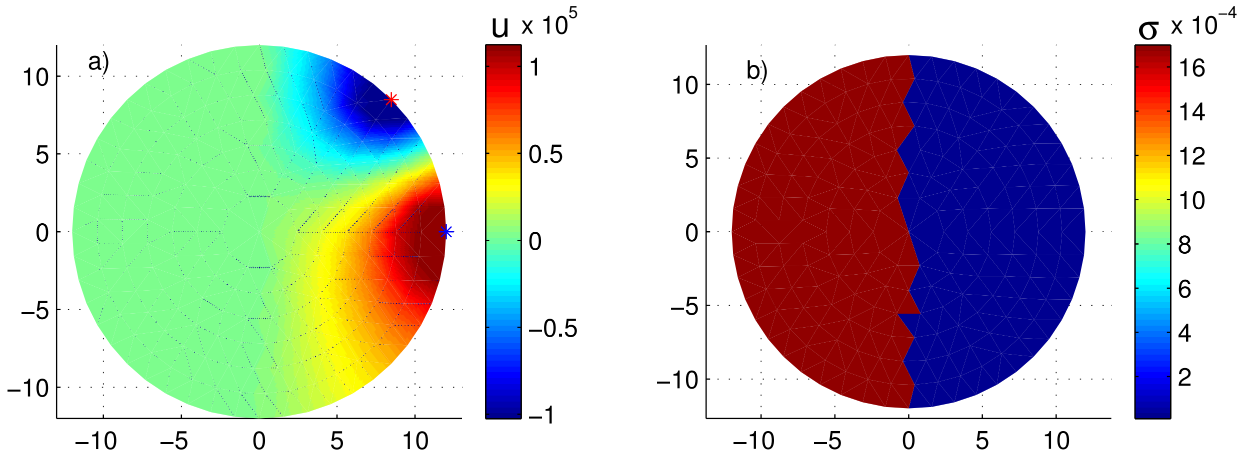

Figure 3: a) potential, interpolated representation; b) distribution of conductivity

Reconstruction-methods applied to real data with large data-error

Problem:

- full-modell algorithms which shows bad results on our real data
- heuristic algorithms which shows reasonable (not good) results on our real data

Our goal: Find full-modell algorithms which deliver reasonable or even good results for the real data under consideration.

We tested 2 known heuristic and 1 known full-modell algorithms on data from our dummy to solve the inverse problem.

Only the heuristic approaches showed reasonable results on our real data with an high error of approx 10 %.

With this we developed another full-model algorithm.

Simple layer backprojection

We used 2 backprojection-based models which in principle are based on the linearity between impedance and voltage (like Ohm's law) with a geometry-dependent factor k.

$$\frac{1}{\sigma} = \rho = k\frac{U}{I}$$

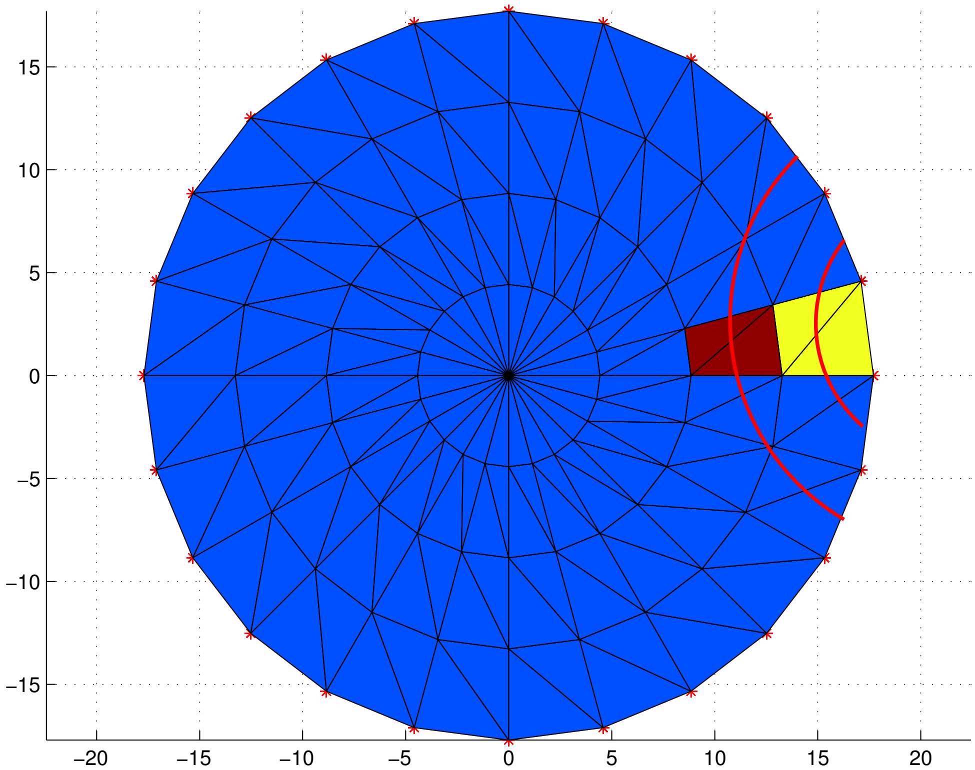

Figure 4: used grid and visualization of idea

Tomographic backprojection

- assume straight lines between the two pairs of electrodes.
- each triangle is representing a resistor and use a series connection which leads to an equation system which can be solved using least-square:

N = number of triangles, ρ known , σ_j unknown and $j = 1, \ldots, N$
Example with 4 rectangles:

$$\rho_{12} = \frac{1}{\sigma_1} + \frac{1}{\sigma_2}$$
$$\rho_{13} = \frac{1}{\sigma_1} + \frac{1}{\sigma_3}$$
$$\vdots$$

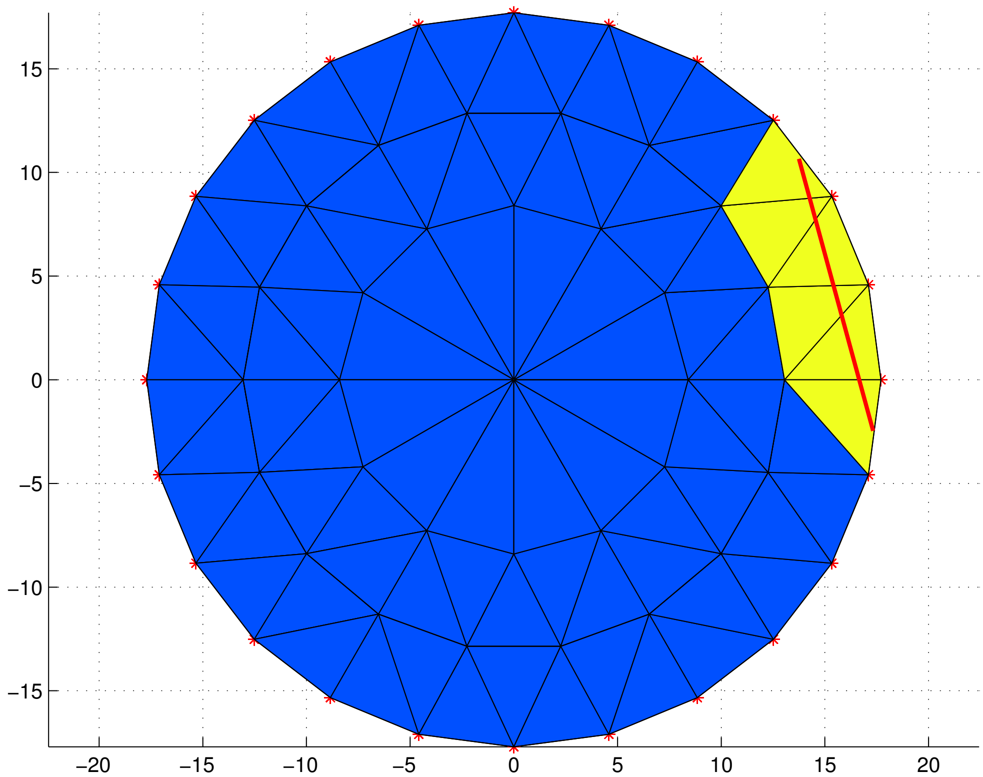

Figure 5: used grid and visualization of idea

Method of successive local least-square

large data error of approx 10 % $\Rightarrow$ an algorithm is needed, which is strongly regularized but still shows good results.

1. one step consists of fitting σ in one element, let others be constant. fit over only 3 possible values.
2. repeat (1) with updated σ until all elements are reached.
3. repeat (1) and (2) until result ist satisfying.

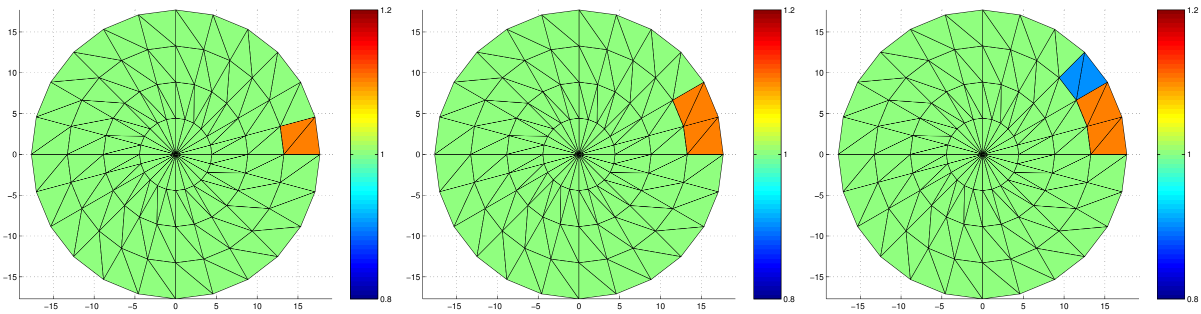

Figure 6: example first steps of algorithm

the regularization is done through this successive projection on low-dimensional subspaces and the strong discretization of the least-square fitting.

Real test-situation

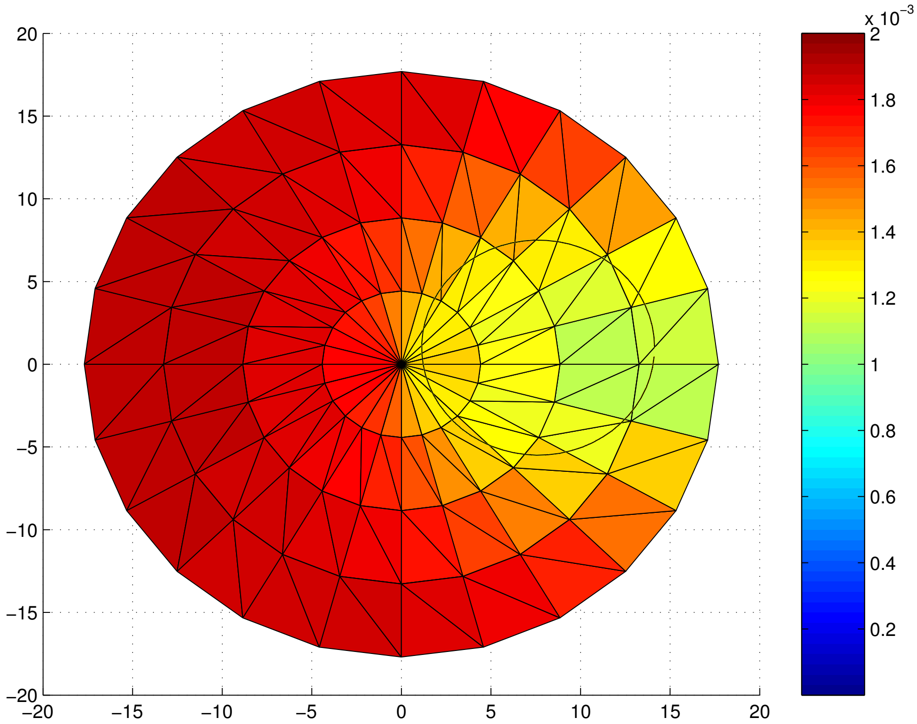

Figure 7: reconstruction with simple layer backprojection

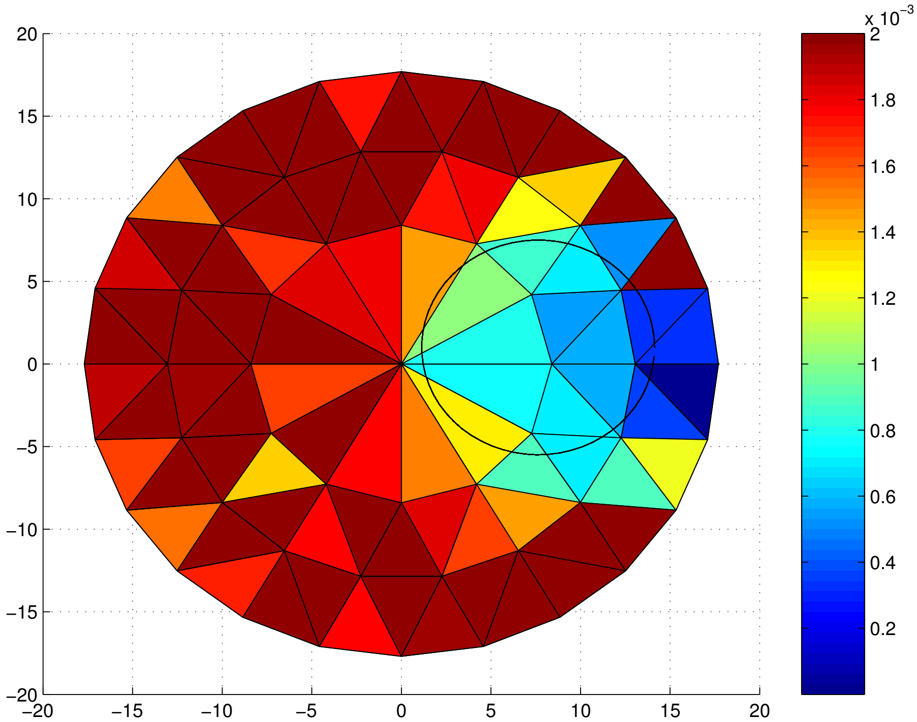

Figure 8: reconstruction with tomographic backprojection

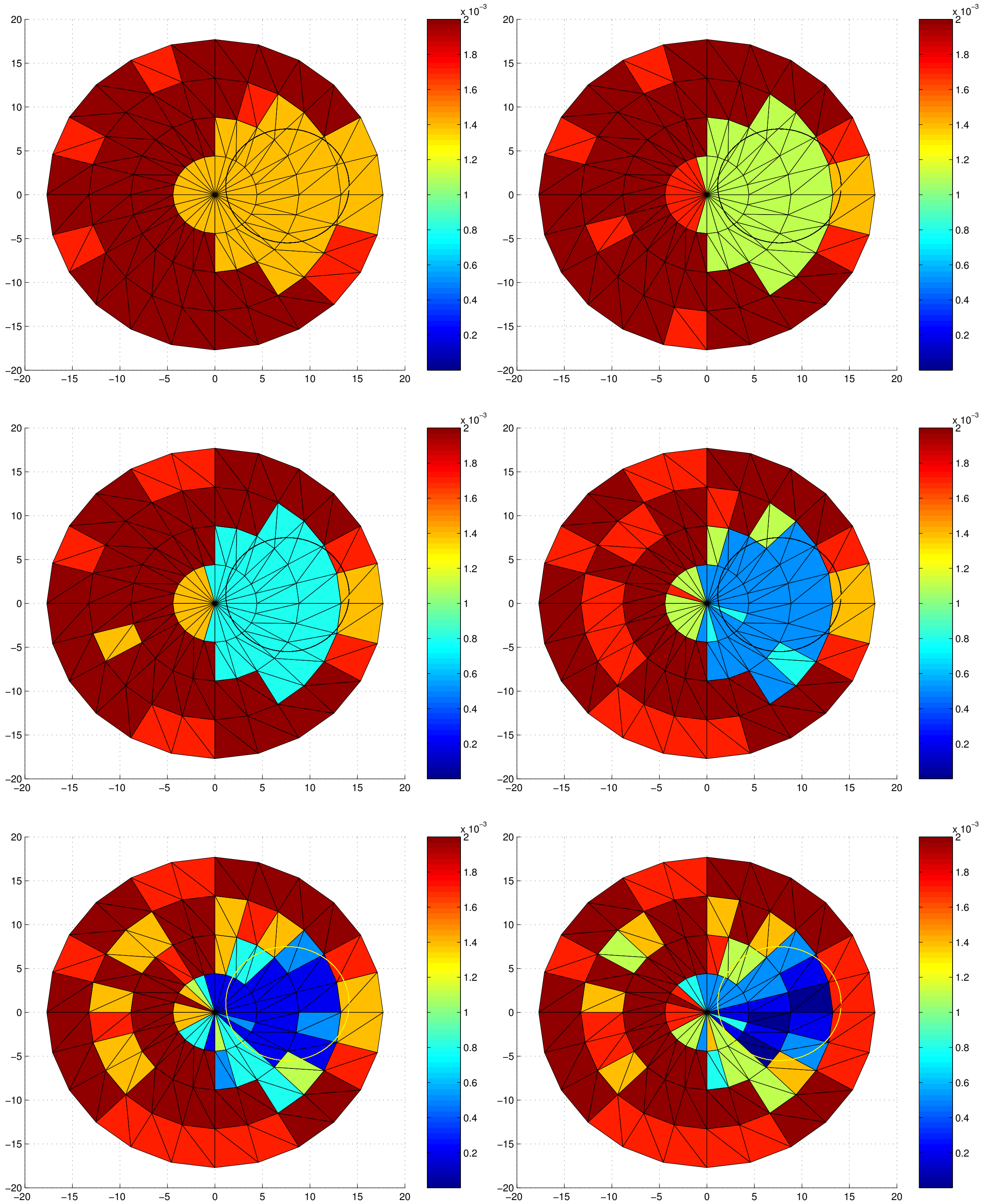

Figure 9: reconstruction with our least-square method on iterations 1 to 6