WS 2002/2003
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2. Exercise in Mathematics and Scientific Computing for Inverse Problems

(numerical integration and differentiation)

1. Problem (2 points)

Write a function in Matlab which numerically differentiates a function in 1 dimension. Add random gaussian noise to the function. Chose different (large and small) stepsize h for calculating the derivative and plot the analytical derivative and the computed numerical derivative of the function.

2. Problem (4 points)

Consider a rectangle $D \subset \mathbb{R}^2$. Let $\Gamma_1 \subset D$ be a vertical line at $x = 0$. Write a function in Matlab which calculates

$$u = (A\varphi)(x) = \int_{\partial\Gamma_1} k(x,y)\varphi(y)\,\mathrm{d}\,y \quad , \; x \in D \backslash \partial\Gamma_1$$

with $k(x,y) = \ln\left(|x-y|^2\right)$ and $\varphi(y) = 1$. Calculate and plot numerical and analytical derivative of u.

3. Problem (2 points)

Consider the same situation as in problem 2. Further let $\Gamma_2 \subset D$ be a vertical line at some $x \neq 0$. Construct a matrix for the operator $A : C(\Gamma_1) \to C(\Gamma_2)$ and plot the result $A\varphi = f$. Then invert the matrix $A^{-1} : C(\Gamma_2) \to C(\Gamma_1)$ and plot the result of $\varphi_r = A^{-1}f$. Compare φ and φ_r and calculate the norms of A and A^{-1}.

4. Problem (4 points)

Show that an integral operator with a weakly singular kernel is a compact operator $D \subset \mathbb{R}^n \to D \subset \mathbb{R}^n$ with D be a nonempty compact and Jordan measureable set.

Sum of Points: 12