WS 2002/2003 October 28, 2002
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http://www.math.uni-goettingen.de/schulz/

3. Exercise in Mathematics and Scientific Computing for Inverse Problems

(ill-posedness, typical equations and their problems)

1. Problem (2 points)

Write a Matlab function which performs a deconvolution with fast fourier transform on

$$G_n := \left\{ -a + \frac{2a}{n} j, j = 1, \ldots, n \right\}$$

Solve $k * \varphi = f$ with a gaussian kernel $k(\xi) := e^{-\tau\xi^2}$ for some f with noise and without. Calculate the norms and plot the behaviour of φ.

2. Problem (3 points)

Transform the helmholtz equation in $\mathbb{R}^2$ in polar coordinates. Show that $u(x) = f(kr)e^{\pm in\varphi}$ solves the helmholtz equation if f is a solution to the Bessel differential equation. Show that

$$J_n(t) = \sum_{p=0}^{\infty} \frac{(-1)^p}{p!(n+p)!} \left(\frac{t}{2}\right)^{n+2p}$$

and

$$Y_n(t) = \frac{2}{\pi} \left\{ ln\left(\frac{t}{2}\right) + C \right\} J_n(t) - \frac{1}{\pi} \sum_{p=0}^{n-1} \frac{(n-1-p)!}{p!} \left(\frac{2}{t}\right)^{n-2p} -$$
$$\frac{1}{\pi} \sum_{p=0}^{\infty} \frac{(-1)^p}{p!(n+p)!} \left(\frac{t}{2}\right)^{n+2p} \left\{ \sum_{m=1}^{p+n} \frac{1}{m} + \sum_{m=1}^{p} \frac{1}{m} \right\}$$

are solutions to the bessel differential equation.

3. Problem (4 points)

Consider the inverse problem of the heat equation as shown in the lecture. Formulate the inverse problem as integral equation of the first kind and determine the spectral representation.

4. Problem (3 points)

Prove the Riemann-Lebesgue Lemma

$$\int_0^\pi K(\cdot, y) \sin(ny) \,\mathrm{d}\, y \to 0, \quad n \to \infty$$

in the mean square norm for a kernel $K \in L^2([0, \pi] \times [0, \pi])$. Explain how this result can be used to illustrate the ill-posedness of integral equation of the first kind.

Sum of Points: 12