WS 2002/2003 November 8, 2002
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http://www.math.uni-goettingen.de/schulz/

4. Exercise in Mathematics and Scientific Computing for Inverse Problems

(regularization)

1. Problem (5 points)

Consider again problem 3 of sheet 2 and add a gaussian noise like in problem 1 of sheet 2. Then implement *Thikonov* regularization and plot regularization-error and data-error and find the best regularization parameter for given error level δ. Then implement the discrepancy principle, an a posteriori strategy to find an appropriate regularization parameter.
The discrepancy principle is the following:

$$||AR_\alpha f^\delta - f^\delta|| = \gamma\delta$$

with some fixed $\infty >> \gamma \geq 1$. There will be more information to this in the exercise course before delivery.

2. Problem (3 points)

Consider problem 1. Now calculate a singular value decomposition of the operator (Matlab-function: `svd`) and implement a *spectral cut-off* regularization. Compare results of all regularizations.

3. Problem (3 points)

Consider problem 1. Implement the *Landweber-Fridmann* iteration as regularization sheme and compare results of all implemented regularizations.

4. Problem (4 points)

Consider the integral operator $A : L^2[0,1] \to L^2[0,1]$ defined by

$$(A\varphi)(x = \int_0^x \varphi(y)\,\mathrm{d}\,y)$$

with $x \in [0,1]$ for which the singular value system ist given by:

$$\mu_n = \frac{2}{(2n-1)\pi}\,, \qquad \varphi_n = \sqrt{2}\cos\frac{(2n-1)\pi x}{2}$$

$$g_n(x) = \sqrt{2}\sin\frac{(2n-1)\pi x}{2}$$

Then consider the central difference quotient

$$(R_h f)(x) := \frac{1}{h}\left\{f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)\right\}$$

for functions f which are odd with respect to $x = 0$ and even with respect to $x = 1$. Show that $R_h, h > 0$ can be interpreted as regularization sheme for A.
(Hint: use fourier series for f)

Sum of Points: 15