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http://www.math.uni-goettingen.de/schulz/

6. Exercise in Mathematics and Scientific Computing for Inverse Problems

(harmonic, boundary value problems)

1. Problem (3 points)

(*mean value theorem*)
Let u be a harmonic in an open ball $B(x; r) := \{y \in \mathbb{R}^m : |y - x| < r\}$ with boundary $\Omega(x; r)$ and continous in the closure $B[x; r]$. Show that then the following holds

$$u(x) = \frac{m}{w_m r^m} \int_{B[x;r]} u(y) \,\mathrm{d}\, y = \frac{1}{w_m r^{m-1}} \int_{\Omega(x;r)} u(y) \,\mathrm{d}\, s(y)$$

(Hint: the following holds on $\Omega(x; r)$: $\operatorname{grad}_y \Phi(x, y) = \frac{\nu(y)}{w_m r^{m-1}}$)

2. Problem (4 points)

(*maximum-minimum principle*)
Show that a harmonic function in a bounded domain D which is continous in $\bar{D}$ attains both its maximum and minimum on the boundary.
(Hint: use the mean value theorem to show that a harmonic function in a domain cannot attain its maximum or minimum unless it is constant.)

DEFINITION 1. (Interior Dirichlet problem)
Find a function u that is harmonic in the bounded domain D of class C^2, is continous in $\bar{D}$ and satisfies the boundary condition

$$u = f \quad \text{on } \partial D$$

where f is a given continous function.

DEFINITION 2. (Exterior Dirichlet problem)
Find a function u that is harmonic in $\mathbb{R}^m \setminus \bar{D}$, is continous in $\mathbb{R}^m \setminus D$ and satisfies the boundary condition

$$u = f \quad \text{on } \partial D$$

where f is a given continous function. For $|x| \to \infty$ it is required that for $m = 2$, $u(x) = O(1)$ and for $m = 3$, $u(x) = o(1)$.

3. Problem

(4 points)

- Show that the interior and exterior dirichlet problem has at most one solution.
- Consider exterior Dirichlet problem and define a bounded domain $B \subset \mathbb{R}^2$ of class C^2 which contains the closure of D. Then consider inverse problem of finding boundary values of u on ∂D from given $u|_{\partial B}$. Show that this problem has at most one solution.

Sum of Points: 11