WS 2002/2003 January 9, 2003
Roland Potthast
Jochen Schulz
http://www.math.uni-goettingen.de/schulz/
Delivery: 17th jan. 14:00

8. Exercise in Mathematics and Scientific Computing for Inverse Problems

(Direct acoustic obstacle scattering)

1. Problem (6 points)

Consider the 2 dimensional direct acoustic obstacle scattering problem on a sound soft obstacle, i.e. with dirichlet boundary condition. Let the obstacle domain D be the unit circle in the origin and the incident field be a plane wave $u^i(x) = e^{ikx\cdot d}$ with unit vector d describing the direction of the incoming wave. Numericly solve the problem using surface potentials using an Ansatz for u with an combined single layer and double layer potential in the following form:

$$u(x) = \int_{\partial D} \left\{ \frac{\partial \Phi(x,y)}{\partial \nu(y)} - i\eta\Phi(x,y) \right\} \varphi(y) \, \mathrm{d}\, s(y)$$

whereas $\varphi \in C(\partial D)$ and $\eta \neq 0$ a real coupling parameter. Use rectangular or trapezoidal rule for integration. Plot both scattered and total field.

2. Problem (3 points)

Test the correctness of the program from problem 1 with a known exact solution. For the test use boundary data f determined through a point source in the origin instead of the plane wave.

3. Problem (4 points)

Calculate the far field pattern u_∞ of the solution of problem 1. In 2 dimensions the far field pattern is given by:

$$u_\infty(\hat{x}) = \frac{e^{i\pi/4}}{\sqrt{8\pi k}} \int_{\partial D} \left\{ u(y) \frac{\partial e^{-ik\hat{x}\cdot y}}{\partial \nu(y)} - \frac{\partial u}{\partial \nu}(y) e^{-ik\hat{x}\cdot y} \right\} \mathrm{d}\, s(y)$$

Show the behaviour of u_∞ in dependance to n, the number of sampling points, so get an impression of the order of convergence.

Sum of Points: 13